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The World System Urbanization Dynamics: A quantitative analysis[1]

Andrey Korotayev

The available estimates of the World System[2] urban population up to 1990 may be plotted graphically in the following way (see Diagram 1):

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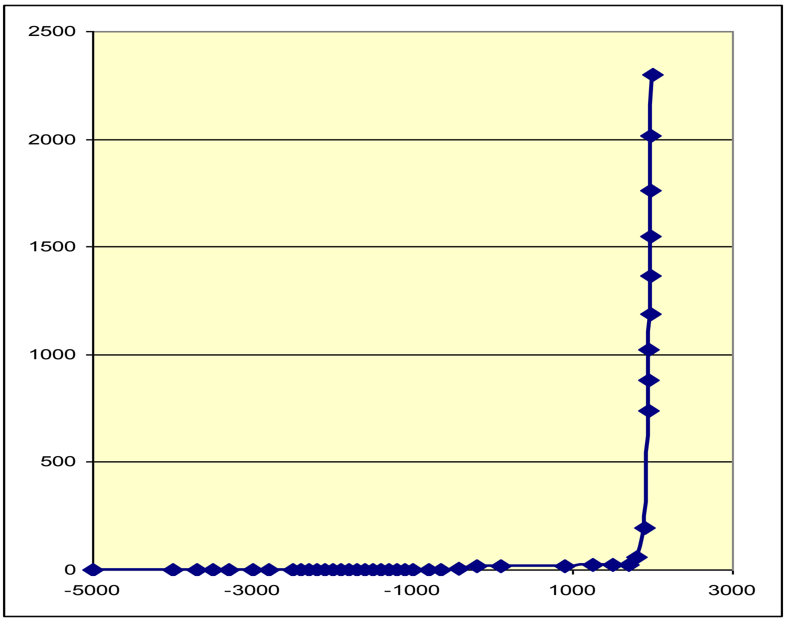
[2] We are speaking here about the system that originated in the early Holocene in the Middle East in direct connection with the start of the Agrarian ("Neolithic") revolution, and that eventually encompassed the whole world. With Andre Gunder Frank (1990, 1993) we denote this system as "the World System". As we have shown (Korotayev, Malkov, and Khaltourina 2006a, 2006b), this was the World System development that produced the hyperbolic trend of the world's population growth. The presence of a hyperbolic trend itself indicates that the major part of the respective entity (that is, the world population in our case) had a systemic unity; and we believe that the evidence for this unity is readily available. Indeed, we have evidence for the systematic spread of major innovations (domesticated cereals, cattle, sheep, goats, horses, plow, wheel, copper, bronze, and later iron technology, and so on) throughout the whole North African – Eurasian Oikumene for a few millennia BCE (see, *e.g.*, Чубаров 1991, or Diamond 1999 for a synthesis of such evidence). As a result, the evolution of societies in this part of the world, already at this time, cannot be regarded as truly independent. Note, of course, that there would be no grounds for speaking about a World System stretching from the Atlantic to the Pacific, even at the beginning of the 1st millennium CE, if we applied the "bulk-good" criterion suggested by Wallerstein (1974, 1987, 2004), as there was no movement of bulk goods at all between, say, China and Europe at this time (as we have no reason to disagree with Wallerstein in his classification of the 1st century Chinese silk reaching Europe as a luxury rather than a bulk good). However, the 1st century CE (and even the 1st millennium BCE) World System definitely qualifies as such if we apply the "softer" information-network criterion suggested by Chase-Dunn and Hall (1997). Note that at our level of analysis the presence of an information network covering the whole World System is a perfectly sufficient condition, which makes it possible to consider this system as a single evolving entity. Yes, in the 1st millennium BCE any bulk goods could hardly penetrate from the Pacific coast of Eurasia to its Atlantic coast. However, the World System had reached by that time such a level of integration that iron metallurgy could spread through the whole of the World System within a few centuries. Another important point appears to be that even by the 1st century CE the World System had encompassed appreciably less than 90 per cent of all the inhabitable landmass. However, it appears much more important that already by the 1st century CE more than 90% of the world population lived precisely in those parts of the world that were integral parts of the World System (the Mediterrannean region, the Middle East, as well as South, Central, and East Asia) (see, *e.g.*, Durand 1977: 256), whereas almost all the urban population of the world was concentrated just within the World System. A few millennia before, we would find another belt of societies strikingly similar in level and character of cultural complexity, stretching from the Balkans up to the Indus Valley outskirts, that also encompassed most of the world population of that time (Peregrine and Ember 2001: vols. 4 and 8; Peregrine 2003). Thus, already for many millennia the dynamics of the world population, the world urbanization, the world political centralization and so on reflect first of all the dynamics of population, urbanization, political centralization, *etc.*, of the World System that makes it possible to describe them with mathematical macromodels.

Diagram 1. Dynamics of the World Urban Population (millions), for cities with > 10000 inhabitants (5000 BCE – 1990 CE)



NOTES. Data sources: Modelski 2003; Gruebler 2006; UN Population Division 2006. Modelski provides his estimates of the world urban population (for cities with >10000 inhabitants) for the period till 1000 BCE, Gruebler's estimates cover the period between 900 and 1950 CE, whereas the UN's estimates cover the period after 1950. The estimates of the world urban population for the period between 1000 BCE and 900 CE were produced on the basis of Chandler's (1987) data on the world urban population living in large cities (with >40000 inhabitants).

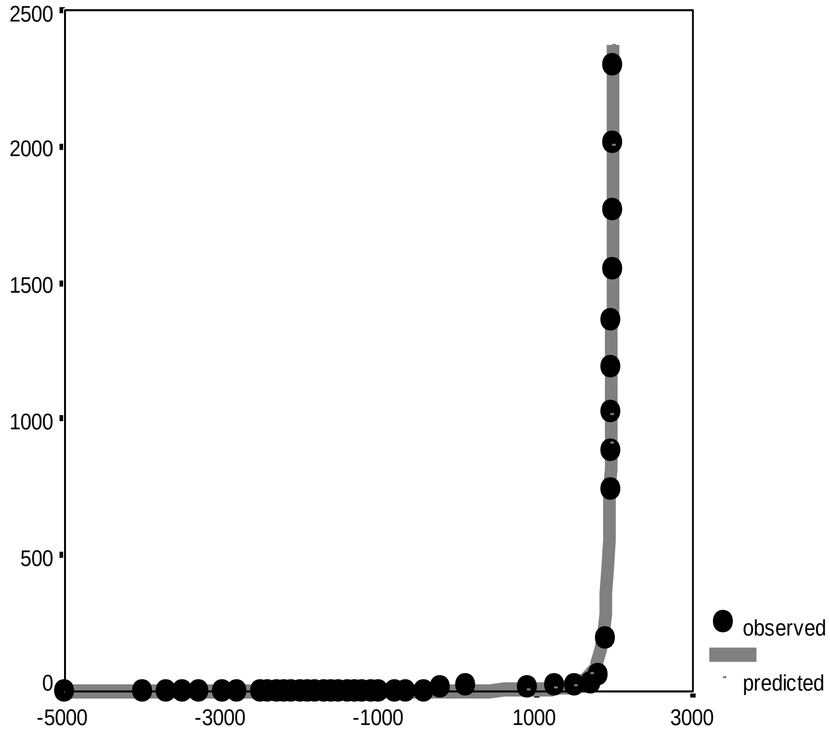
As has been shown by us earlier (see, *e.g.*, Коротаев 2006; Коротаев, Малков, Халтурина 2006; Korotayev, Malkov, and Khaltourina 2006b), the overall dynamics of the world urban population up to the 1990s are described mathematically in a rather accurate way by the following quadratic-hyperbolic equation:

$$U_t = \frac{C}{(t_0 - t)^2}, \qquad (1)$$

where U_t is the world urban population in the moment t, whereas C and t_0 are constants, with t_0 corresponding to an absolute limit ("singularity" point) at which U would become infinite if the world urban population growth trend observed by the 1990s continued further.

Thus, for the period between 5000 BCE and 1990 CE the correlation between the dynamics generated by equation (1) and empirical estimates looks as follows (see Diagram 2):

Diagram 2. World Urban Population Dynamics (in millions), for cities with > 10000 inhabitants (5000 BCE – 1990 CE): correlation between the dynamics generated by the quadratic-hyperbolic model and empirical estimates



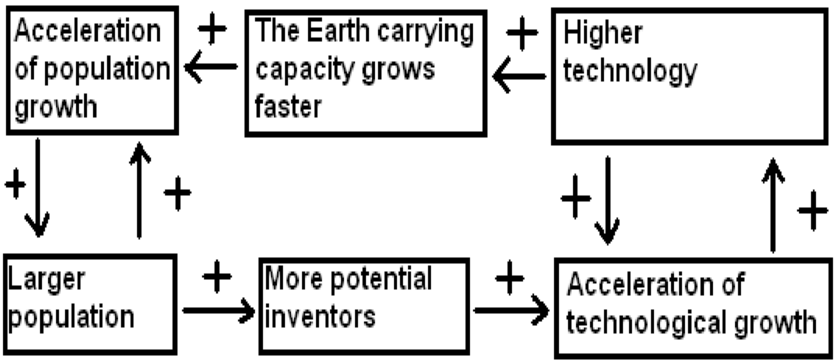
NOTES: $R = 0.998$, $R^2 = 0.996$, $p << 0.0001$. Black markers correspond to empirical estimates of Modelski (2003), Gruebler (2006) and UN Population Division (2006). The solid grey curve was generated by the following equation:

$$U_t = \frac{7705000}{(2047 - t)^2}.$$

Parameters C (7705000) and t_0 (2047) have been calculated with the least squares method.

The observed very high level of correlation between the long-term macrodynamics of the world urban population and the dynamics generated by the quadratic-hyperbolic model does not seem coincidental at all and is accounted for by the presence of second-order nonlinear positive feedback loops between the world's demographic growth and the World System technological development that can be spelled out as follows: the more people – the more potential inventors – the faster technological growth – the faster growth of the Earth's carrying capacity – the faster population growth – with more people you also have more potential inventors – hence, faster technological growth, and so on (Kuznets 1960; Simon 1977, 1981, 2000; Grossman and Helpman 1991; Aghion and Howitt 1992, 1998; Jones 1995, 2003, 2005; Kremer 1993; Cohen, 1995; Komlos and Nefedov 2002; Подлазов 2000, 2001, 2002; Podlazov, 2004; Tsirel 2004; Коротаев, Малков, Халтурина 2005, 2006; Korotayev, Malkov, and Khaltourina 2006a, 2006b) (see Diagram 3):

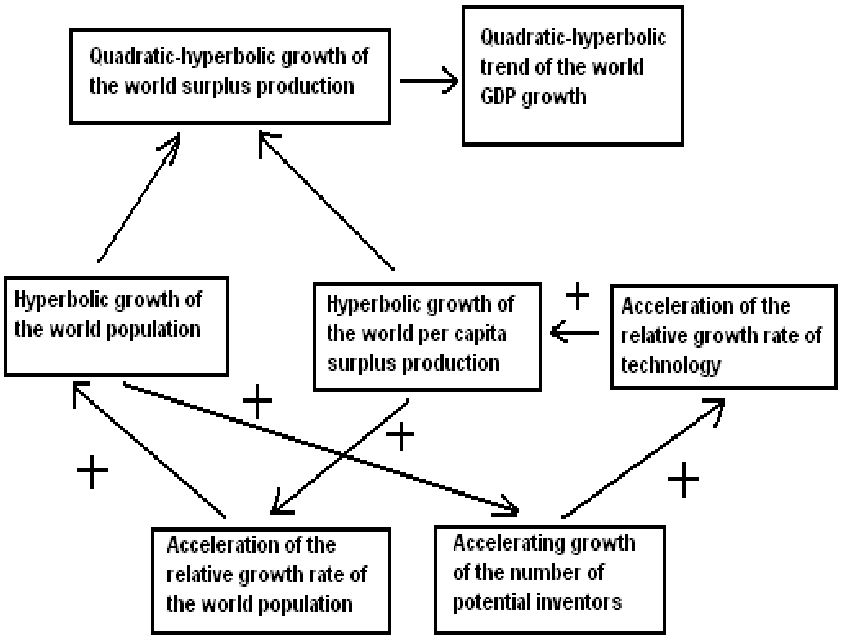
Diagram 3. Block Scheme of the Nonlinear Second Order Positive Feedback between Technological Development and Demographic Growth



As our (both mathematical and empirical) analysis (see, *e.g.*, Коротаев, Малков, Халтурина 2005a, 2005b; Korotayev, Malkov, and Khaltourina 2006a) suggests, up to the 1970s the above mentioned mechanism tended to lead not only to the hyperbolic growth of the World System population, but also to the hyperbolic growth of the per capita surplus[3] and also to the quadratic-hyperbolic growth of the world GDP (see Diagram 4):

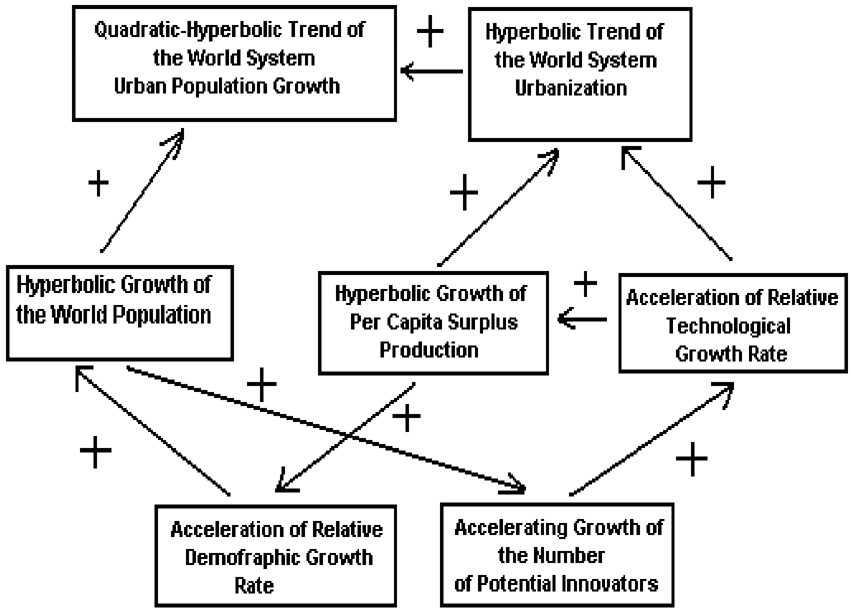
[3] That is, the product produced, per person, over the amount (m) minimally necessary to reproduce the population with a zero growth rate in a Malthusian system.

Diagram 4. Block Scheme of the Nonlinear Second Order Positive Feedback between Technological Development, Demographic and Economic Growth



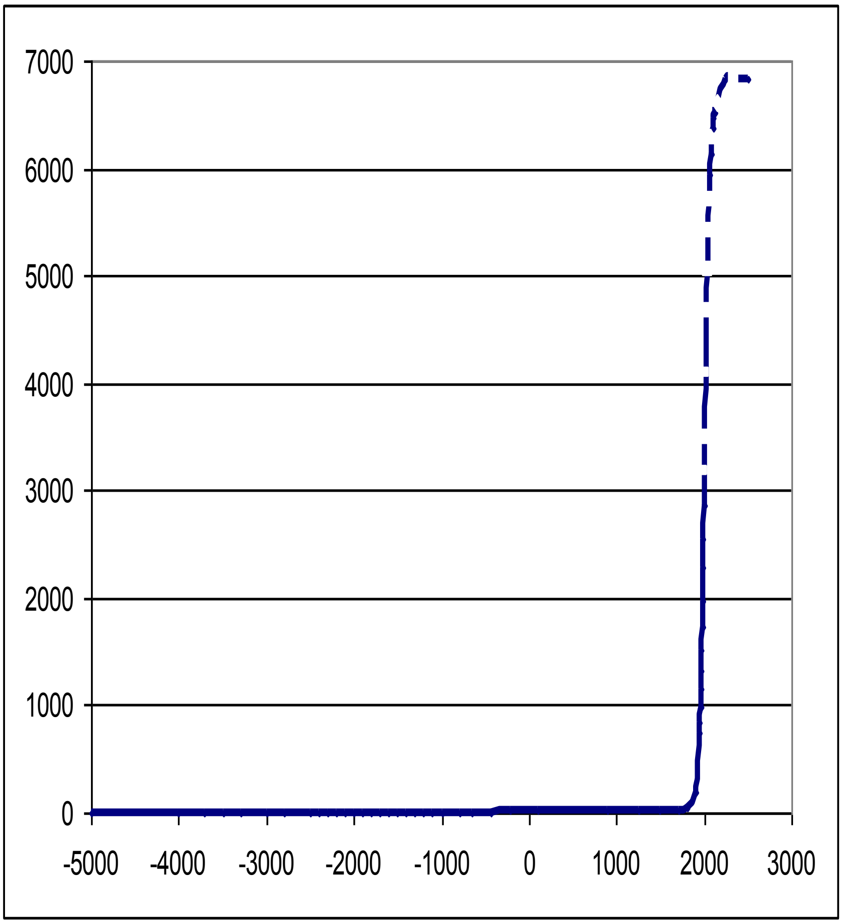
Up to the 1970s – 1990s the trend towards the hyperbolic growth of the per capita surplus production (in conjunction with the hyperbolic acceleration of the technological growth) tended to result in the trend towards the hyperbolic growth in world urbanization (that is, the hyperbolic growth of the proportion of the urban population in the total population of the world); in conjunction with the hyperbolic growth of the world's population this, naturally, also produced a long-term trend towards the quadratic-hyperbolic growth of the world urban population (see Diagram 5):

Diagram 5. Block Scheme of the Nonlinear Second Order Positive Feedback Generating the Trend towards the Quadratic-Hyperbolic Growth of the World System Urban Population



The best fit of the dynamics generated by the quadratic-hyperbolic equation (1) to the empirical estimates of the world urban population is observed for the period prior to 1965. For this period, equation (1) describes more than 99.88% of all the macrovariation of the variable in question (R = 0.9994, R^2 = 0.9988, with the following parameter values: C = 2610000 [in millions], t_0 = 2010). Incidentally, the above mentioned parameter value (t_0 = 2010 [CE]) indicates that if the world urban population growth trend observed prior to the mid 1960s continued, the world urban population would become infinite already in 2010. That is why, it is hardly surprising that since the mid 1960s the World System started its withdrawal from the blow-up regime with respect to the variable in question. Indeed, since the 1960s we observe the slow-down of the relative rate of the world urban population growth, and, according to the forecasts (see, *e.g.*, Gruebler 2006) in the forthcoming decades the slow-down of the absolute rates of the world population growth will also start, resulting in the stabilization of the world urban population in the 22nd century at the level of about 7 billion (see Diagram 6):

Diagram 6. World Urban Population Dynamics (in millions), for cities with >10000 inhabitants (5000 BCE– 2006 CE), with a forecast till 2350



NOTES. Data sources: Modelski 2003; Gruebler 2006; UN Population Division 2006. The curve for 2006–2350 has been calculated on the basis of Gruebler's medium forecast for the dynamics of the world urbanization (*i.e.*, the proportion of the urban population in the overall population of the world) and our own forecast of the world population for this period (Korotayev, Malkov, Khaltourina 2006a).

The general macrodynamics of the World System urbanization can be described mathematically with the following differential equation:

$$\frac{du}{dt} = aSu(u_{\lim} - u) , \quad (2)$$

where u is the proportion of the population that is urban, S is per capita surplus produced with the given level of the World System's technological development, a is a constant, and $u_{\lim}$ is the maximum possible proportion of the population that can be urban (that may be estimated as being within 0.8–0.9, and can be regarded within the given context as the "saturation level").

With low values of u (< 0.3) its dynamics is determined first of all by the hyperbolic growth of S,[4] as a result of which the urbanization dynamics turn out to be also close to the hyperbolic dynamics, which, in conjunction with the hyperbolic growth of the World System population (that was naturally observed just for the period characterized by low values of the world urbanization) led to the fact that the overall macrodynamics of the world urban population for this period was described very well by the quadratic-hyperbolic equation. With higher values of the world urbanization index (u) the saturation effect begins being felt more and more strongly, and as it approaches saturation level the world urbanization growth rates begin to slow down more and more, which is observed at present – a time when the World System has begun its withdrawal from the blow-up regime.

It is difficult not to notice that the history of world urbanization up to the 19th century looks, in Diagrams 1–2 and 6, extremely "dull", producing an impression of an almost perfect stagnation[5] followed by explosive modern urban population growth. In reality the latter just does not let us discern, in the diagrams above, the fact that many stretches of the pre-modern world urban history were characterized by dynamics that were comparatively no less dramatic. In fact, the impression of the pre-modern urban stagnation created by diagrams above could be regarded as an illusion (in the strict sense of this word) produced just by the quadratic-hyperbolic trend of the world urban population growth observed up to the mid 1960s. To see this it is sufficient to consider Diagram 1 in a logarithmic scale (see Diagram 7):

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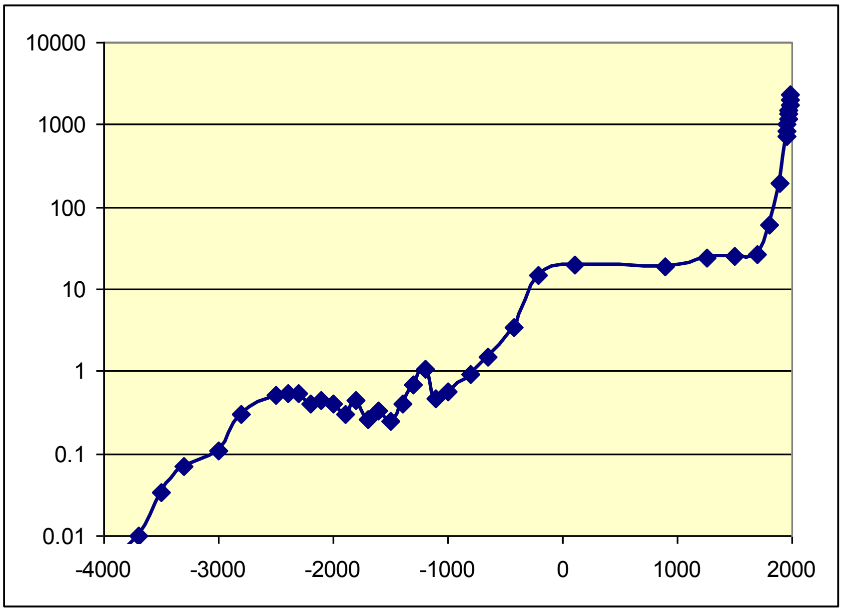
[4] For the systems of equations describing this hyperbolic growth generated by the second-order nonlinear positive feedback loops between the World System technological development and the world demographic growth see, *e.g.*, Korotayev, Malkov, and Khaltourina 2006a, 2006b.

[5] Whereas for the period prior to 1000 BCE this stagnation looks absolute.

Diagram 7. Dynamics of the World Urban Population (in millions), for cities with >10000 inhabitants (5000 г. до н.э. – 1990 г. н.э.), LOGARITHMIC SCALE

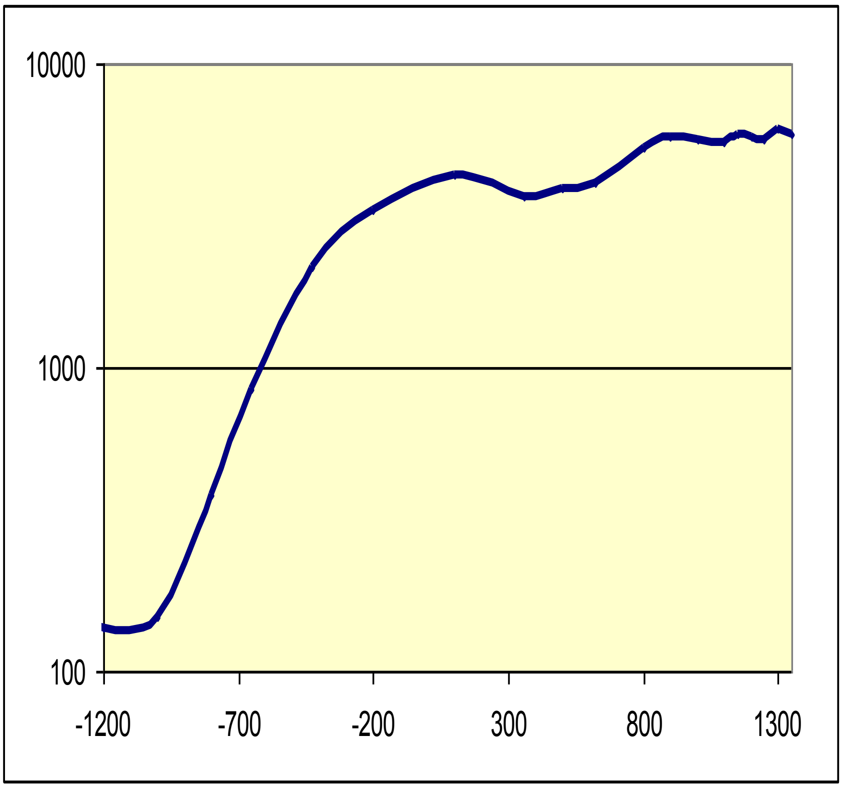


As we see, the structure of the curve of the World System urban population growth turns out to be much more complex than one would imagine at first glance at Diagrams 1–2 and 6. First of all, one can single out in a rather distinct way three periods of relatively fast world urban population growth: (A1) the second half of the 4th millennium BCE – the first half of the 3rd millennium BCE, (A2) the 1st millennium BCE and (A3) the 19th – 21st centuries CE. In addition to this, one can see two periods of relatively slow growth of the world urban population: (B1) the mid 3rd millennium BCE – the late 2nd millennium BCE and (B2) the 1st – 18th centuries CE. As we shall see below, two other periods turn out to be essentially close to these epochs: Period (B0) immediately preceding the mid 4th millennium (when the world urban population did not grow simply because the cities had not appeared yet and no cities existed on the Earth), and Period (B3) that is expected to begin in the 22nd century, when, according to forecasts, the world urban population will stop again to grow in any significant way (in connection with the World System urbanization reaching the saturation level and the stabilization of the world population) (see, *e.g.*, Gruebler 2006; Korotayev, Malkov, and Khaltourina 2006a, 2006b).

As one can see at Diagram 7, in Period B1 (from the mid 3rd millennium BCE to the early 1st millennium BCE) the world urban population fluctuated at the level reached by the end of the previous period (A1), whereas the trend dynamics carved its way with great difficulties through the dominant cyclical and stochastic dynamics (see, *e.g.*, Modelski 2003; Frank and Thompson 2005; Harper 2007). In Diagram 7 one could hardly discern the cyclical component of the world urban population dynamics during Period B2 (the 1st – 18th centuries CE), which is accounted for by the simple fact that the respective stretch of the diagram has been prepared on the basis of Gruebler's database that provides for this period a very small number of data points that is not sufficient for the detection of the cyclical component of the process under study. Within Period B2 this cyclical component will be more visible if we use Chandler's database, which provides much more data points for this period (Chandler 1987: 460–510)[6] (see Diagram 8):

[6] This database consists of lists of the largest cities of the world for various time points with estimates of the respective cities' population for respective moments of time. Chandler provides estimates for the following time points (numbers in brackets indicate the urban population in thousands, for cities with population not smaller than which the estimates are provided for the respective year; for example, number 20 in brackets after 800 BCE indicates that for 800 BCE Chandler's database provides estimates of the urban population for all the world cities with no less than 20 thousand inhabitants) – 2250 BCE (20), 2000 BCE (20), 1800 BCE (20), 1600 BCE (20), 1360 BCE (20), 1200 BCE (20), 1000 BCE (20), 800 BCE (20), 650 BCE (30), 430 BCE (30), 200 BCE (30) and further for the following years CE: 100 (30), 361 (40), 500 (40), 622 (40), 800 (40), 900 (40), 1000 (40), 1100 (40), 1150 (40), 1200 (40), 1250 (40), 1300 (40), 1350 (40), 1400 (45), 1450 (45), 1500 (45), 1550 (50), 1575 (50), 1600 (60), 1650 (58), 1700 (60), 1750 (68), 1800 (20), 1825 (90), 1850 (116), 1875 (192), 1900 (30), 1914 (455), 1925 (200), 1950 (200) and 1970 (1930). The main problem with the use of Chandler's database within the context of the present study is that it turns out to be impossible to get data on the world urban population dynamics through the simple summation of the populations of the cities covered by Chandler for the respective years. Indeed, with such a simple summation we will obtain, for example, for 1825 a figure indicating the total urban population that lived in that year in cities with no less than 90 thousand inhabitants, for 1850 – for the cities with no less than 116 thousand inhabitants, for 1875 – for the cities with no less than 192 thousand inhabitants, for 1900 – for the cities with no less than 30 thousand inhabitants, for 1914 – for the cities with no less than 455 thousand inhabitants; and such a series of numbers will not supply us with any useful information. On the other hand, of course, if for one year we have at our disposal data on cities with >80 thousand inhabitants, for a second – on cities with >120 thousand, and for a third – on cities with >100 thousand, we can trace the urban population dynamics for cities with >120 thousand inhabitants. However, this does not solve the whole problem. Indeed, when we use Chandler's database with respect to the last centuries, we can only obtain a meaningful dynamic time series for the megacities (>200 thousand inhabitants). However, even with this approach we cannot obtain a general picture of the world urban population dynamics for the whole period covered by Chandler's database (that is, since 2250 BCE), as no such megacities existed before the mid 1st millennium BCE. The longest dynamic time series can be here obtained for the cities with no less than 40 thousand inhabitants (especially in conjunction with Modelski's database). However, in this case we cannot move after 1350 CE. Because of this, when using Chandler's database we will have to utilize the data on the total population of large cities (with no less than 40 thousand inhabitants) for the period between 3300 BCE and 1350 CE (in conjunction with Modelski's data on the period before 2250 BCE)

Diagram 8. Urban Population Dynamics (in thousands), for cities with no less than 40,000 inhabitants (1200 BCE – 1350 CE), logarithmic scale

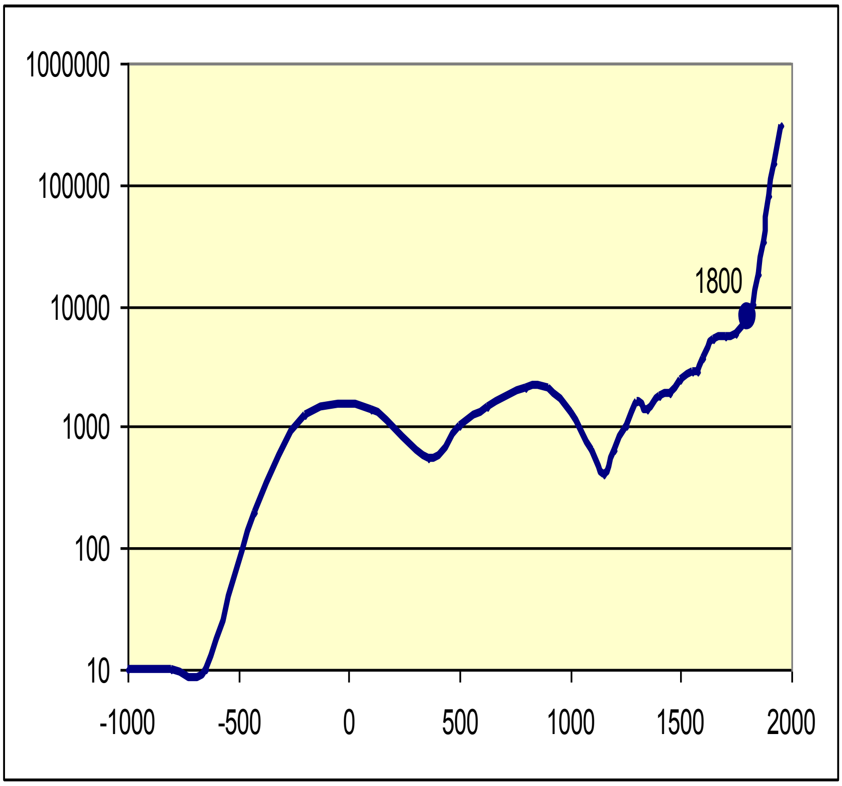


As we see, at this diagram we can observe for Period B2 not only a distinct cyclical component[7], but also a more clear upward trend. This trend will be even more distinctly visible if we plot Chandler's data on population dynamics of megacity (>200,000) inhabitants (which will also make it possible for us to take into account the period after 1350) (see Diagram 9):

and data on the total population of megacities (with no less than 200 thousand inhabitants each) for the period between 430 BCE and 1950 CE.

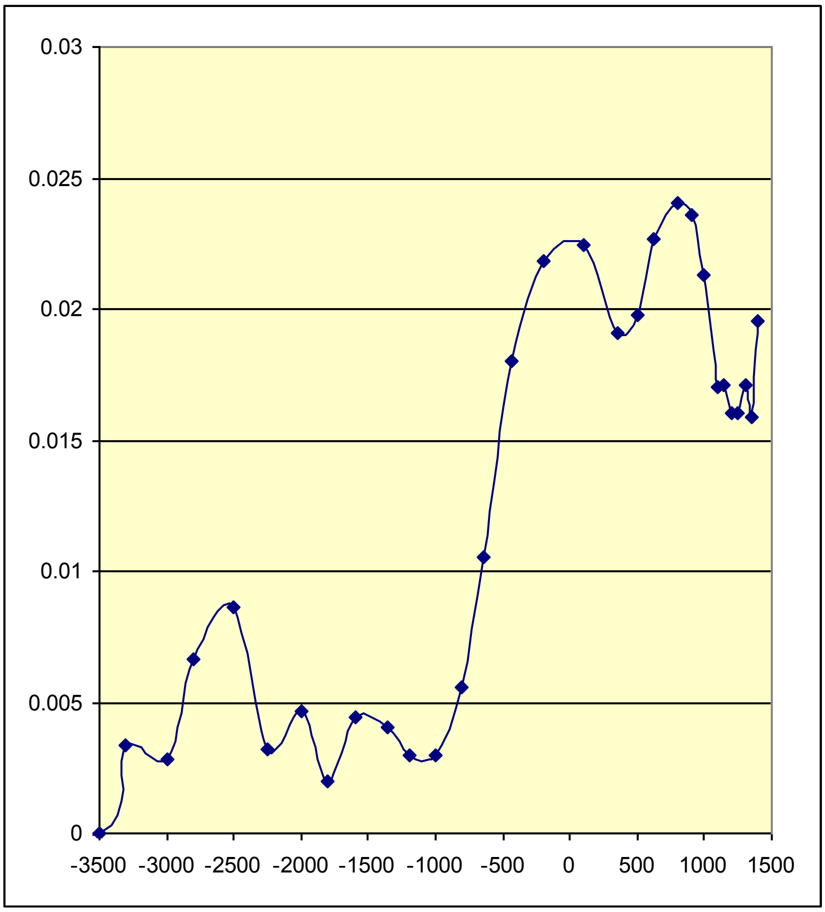
[7] In particular after 1100, which is connected with the point that in Chandler's database after this year the distance between data points get reduced from 100 to 50 years.

Diagram 9. World Urban Population Dynamics (in thousands), for cities with no less than 200,000 inhabitants (1000 BCE – 1950 CE), logarithmic scale



As we see, a steady upward trend can be traced here for a few centuries before 1800. On the other hand, one should take into account the point that a relatively fast growth of the world urban population was observed during this period against the background of the hyperbolically accelerating growth of the overall population of the world (see, *e.g.*, Korotayev, Malkov, and Khaltourina 2006a, 2006b). That is why we shall obtain a clearer picture of the world urbanization dynamics if we plot the estimates of the dynamics of the world urbanization index *per se*, that is the proportion of the urban population in the overall population of the world (see Diagram 10):

Diagram 10. Dynamics of the World Macrourbanization Index (proportion of population living in large, >40000 inhabitants) according to the estimates of Modelski and Chandler (3500 BCE – 1400 CE)

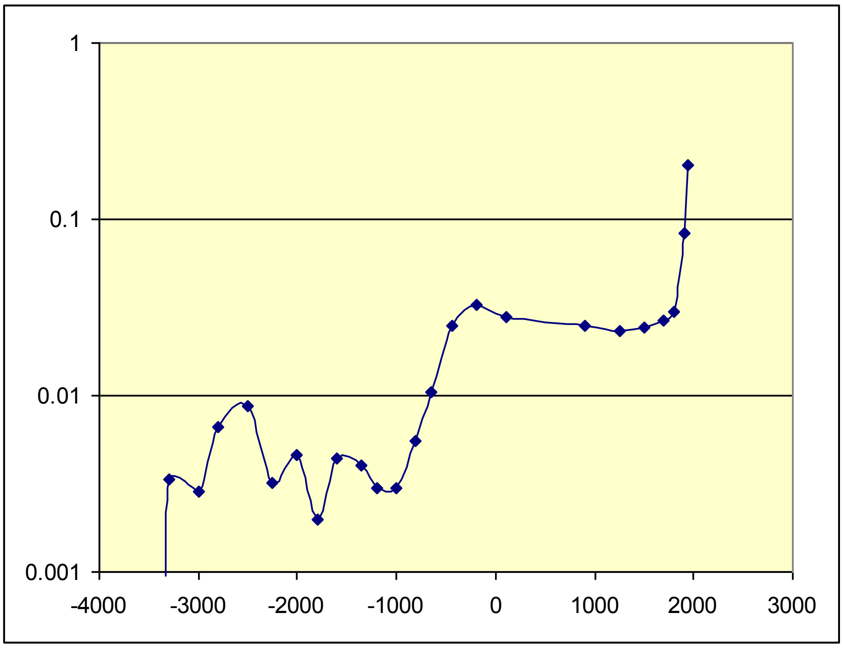


As has been mentioned above, Chandler's database does not make it possible to trace the world macrourbanization dynamics after 1400.[8] That is why in order to obtain an overall picture of the world urbanization dynamics we shall have to rely with respect to Period B2 on Gruebler's estimates (incidentally, let us recollect that because of a very small number of data points in this database the re-

[8] In fact, it produces a bit of a distorted picture already for 1400, as for this year it contains data on the cities with >45 (and not 40) thousand inhabitants.

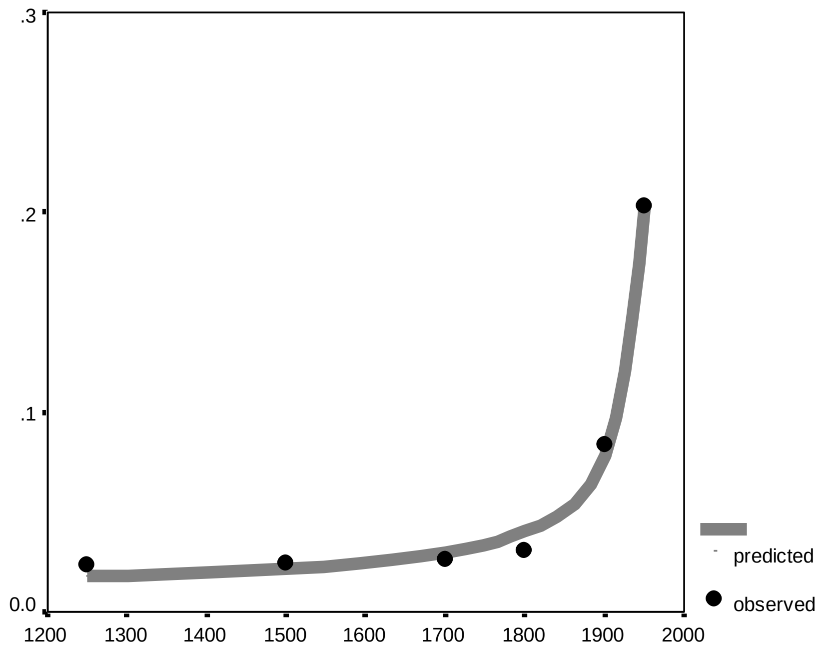
spective graphs do not reflect the cyclical component of the world macro-urbanization dynamics):

Diagram 11. Dynamics of the World Macrourbanization (proportion of population living in large, >40,000, cities in the overall population of the world) according to the databases of Modelski, Chandler, and Gruebler (4000 BCE – 1950 CE), logarithmic scale



Our analysis suggests some idea of the general picture of the long-term macro-urbanization of the world. During Period A1 we observe the formation of the first large cities, and the proportion of their population reached the level of decimals of one per cent of the overall population of the world. During Period B1 this variable had fluctuated within this order of magnitude until, during Period A2, it moved to the further order of magnitude, to the level of more than one per cent. The variable in question had fluctuated within this order of magnitude during Period B2 until, during Period A3, it began its movement to the next (and, note, the last possible) order of magnitude, to the level of dozens per cent. It is also remarkable that for the 2nd millennium CE Gruebler's database indicates a clear hyperbolic trend of the world macro-urbanization described mathematically by model (2) (see Diagram 12):

Diagram 12. World Macrourbanization Dynamics, 1250–1950 CE: correlation between predictions of the hyperbolic model and empirical estimates



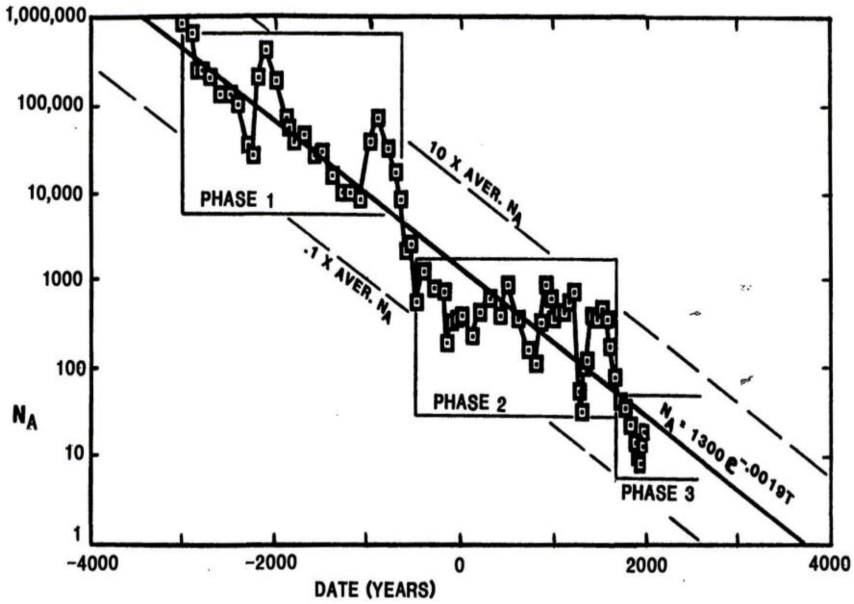
NOTES: R = 0.997, R^2 = 0.994, p < 0.0001. The black markers correspond to Gruebler's (2006) empirical estimates. The solid grey curve has been generated by the following equation:

$$u_t = 0{,}01067 + \frac{5.203}{(1977 - t)}.$$

Parameters C (5,203), t_0 (1977) and the constant (0,01067) have been calculated with the least squares method.

Note that the detected world urbanization dynamics correlates rather well with the dynamics of the World System political organization (see the article by Grinin and Korotayev in the present issue of the Almanac). Note also that the above mentioned synchronous phase transitions to the new orders of magnitude of the world urbanization and new order of the World System political organization complexity coincide in time with phase transitions to higher orders of the World System political centralization that were detected by Taagapera and that took place, according to his calculations, during periods A1, A2 и A3. Taagapera estimates the World System political centralization dynamics using the indicator that he denotes as an "effective number of polities" that is a reverse of the political centralization index (which has values between 0 and 1, where 1 corresponds to the maximum level of the world political centralization, that is the world unification within one polity). Thus, in Diagram 13 below, the downward trend corresponds to the GROWTH of political centralization of the world:

Diagram 13. Dynamics of the "Effective Number of Polities" Calculated on the Basis of Territory Size Controlled by Various Polities (Taagapera 1997: 485, Fig. 4)



Similar phase transitions appear to be observed with respect to the world literacy macrodynamics. Indeed, during Period A1 we observe the appearance of the first literate people whose proportion had reached the level of decimals of one per cent by the end of this period and fluctuated at this level during Period B1. During Period A2. world literacy grew by an order of magnitude and reached the level of several per cent of the total population of the world, it fluctuated at this level during Period B2 till the late 18th century when Period A3 started; during this period the world literacy has reached the level of dozens per cent, and by the beginning of Period B3 (presumably in the 22nd century) it is likely to stabilize at the 100% level (see, *e.g.*, Дьяконов 1994; Мельянцев 1996; Korotayev, Malkov, and Khaltourina 2006a).

In fact, the above mentioned phase transitions can be regarded as different aspects of a series of unified phase transitions: Phase Transition A1 from medium complexity agrarian societies to complex agrarian ones, Phase Transition A2 from complex agrarian societies to supercomplex ones, and, finally, Phase Transition A3 from supercomplex agrarian societies to postindustrial ones (within this perspective, the period of industrial societies turns out to be a period of phase transition B2 – B3).

* * *

Thus, the World System history from the 6th millennium BCE can be described as a movement from Attraction Basin B0 (the one of medium complexity agrarian society) through Phase Transition A1 to Attraction Basin B1 (the one of complex agrarian society), and further through Phase Transition A2 to Attraction Basin B2 (the one of supercomplex agrarian society), and further through Phase Transition A3 to Attraction Basin B3 (the one of postindustrial society). Note that within this perspective the industrial period turns out to be a period of phase transition from the preindustrial society to the postindustrial one.

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