

Introduction: Millennial Trends¹

In the first part of our *Introduction to Social Macrodynamics* (Korotayev, Malkov, and Khaltourina 2006a) we have shown that more than 99% of all the variation in demographic, economic and cultural macrodynamics of the World System over the last two millennia can be accounted for by very simple general mathematical models. Let us start this part with a summary of these findings, along with some relevant new findings that we obtained after the first part of this *Introduction* had been published. This summary is intended for all those interested in patterns of social evolution and development, including those who are not familiar with higher mathematics. Accordingly, we have included some basic material that mathematically sophisticated readers may wish to skip over lightly or entirely ignore.

In 1960 von Foerster, Mora, and Amiot published, in the journal *Science*, a striking discovery. They showed that between 1 and 1958 CE the world's population (N) dynamics can be described in an extremely accurate way with an astonishingly simple equation:²

$$N_t = \frac{C}{t_0 - t}, \quad (0.1)$$

where N_t is the world population at time t , and C and t_0 are constants, with t_0 corresponding to an absolute limit ("singularity" point) at which N would become infinite.

Parameter t_0 was estimated by von Foerster and his colleagues as 2026.87, which corresponds to November 13, 2006; this made it possible for them to

¹ This book is a translation of an amended and enlarged version of the second part of the following monograph originally published in Russian:

(.)., 2005.

² To be exact, the equation proposed by von Foerster and his colleagues looked as follows:

$$N_t = \frac{C}{(t_0 - t)^{0.99}}. \text{ However, as has been shown by von Hoerner (1975) and Kapitza (1992, 1999),}$$

it can be written more succinctly as $N_t = \frac{C}{t_0 - t}$.

supply their article with a public-relations masterpiece title – "Doomsday: Friday, 13 November, A.D. 2026".³

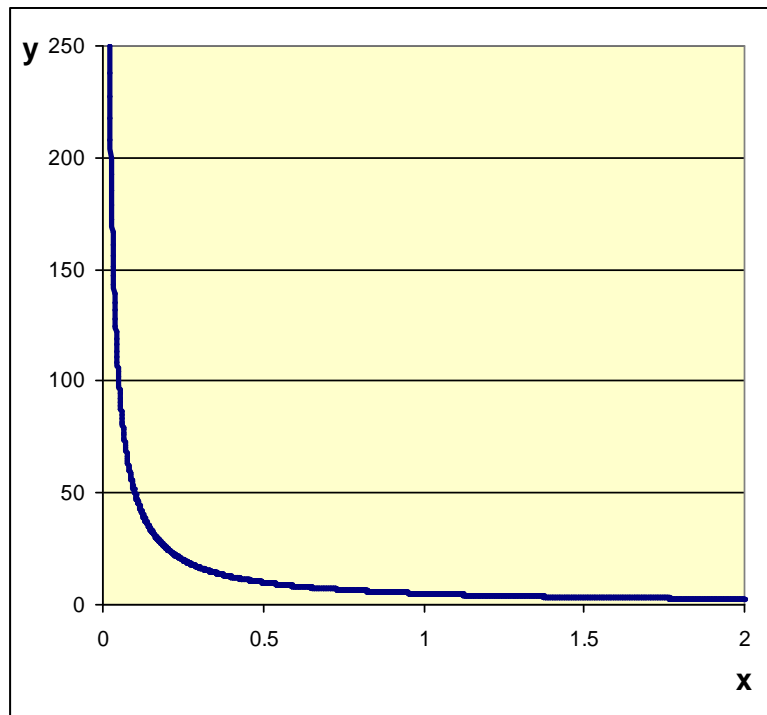
Note that the graphic representation of this equation is nothing but a hyperbola; thus, the growth pattern described is denoted as "hyperbolic".

Let us recollect that the basic hyperbolic equation is:

$$y = \frac{k}{x}. \quad (0.2)$$

A graphic representation of this equation looks as follows (if k equals, *e.g.*, 5) (see Diagram 0.1):

Diagram 0.1. Hyperbolic Curve Produced by Equation $y = \frac{5}{x}$



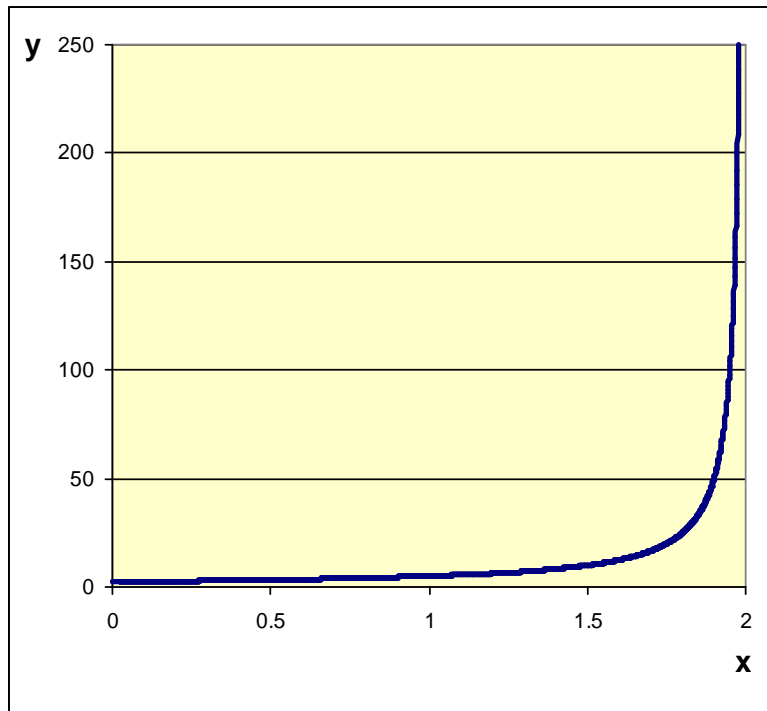
³ Of course, von Foerster and his colleagues did not imply that the world population on that day could actually become infinite. The real implication was that the world population growth pattern that was followed for many centuries prior to 1960 was about to come to an end and be transformed into a radically different pattern. Note that this prediction began to be fulfilled only in a few years after the "Doomsday" paper was published (see, *e.g.*, Korotayev, Malkov, and Khaltourina 2006a: Chapter 1).

The hyperbolic equation can also be written in the following way:

$$y = \frac{k}{x_0 - x} . \quad (0.3)$$

With $x_0 = 2$ (and k still equal to 5) this equation will produce the following curve (see Diagram 0.2):

Diagram 0.2. Hyperbolic Curve Produced by Equation $y = \frac{5}{2 - x}$



As can be seen, the curve produced by equation (0.3) at Diagram 0.2 is precisely a mirror image of the hyperbolic curve produced by equation (0.2) at Diagram 0.1. Now let us interpret the X-axis as the axis of time (t -axis), the Y-axis as the axis of the world's population (counted in millions), replace x_0 with 2027 (that is the result of just rounding of von Foester's number, 2026.87), and re-

place k with 215000.⁴ This gives us a version of von Foerster's equation with certain parameters:

$$N_t = \frac{215000}{2027 - t}. \quad (0.4)$$

In fact, von Foerster's equation suggests a rather unlikely thing. It "says" that if you would like to know the world population (in millions) for a certain year, then you should just subtract this year from 2027 and then divide 215000 by the difference. At first glance, such an algorithm seems most unlikely to work; however, let us check if it does. Let us start with 1970. To estimate the world population in 1970 using von Foerster's equation we first subtract 1970 from 2027, to get 57. Now the only remaining thing is to divide 215000 by the figure just obtained (that is, 57), and we should arrive at the figure for the world population in 1970 (in millions): $215000 \div 57 = 3771.9$. According to the U.S. Bureau of the Census database (2006), the world population in 1970 was 3708.1 million. Of course, none of the U.S. Bureau of the Census experts would insist that the world population in 1970 was precisely 3708.1 million. After all, the census data is absent or unreliable for this year for many countries; in fact, the result produced by von Foerster's equation falls well within the error margins for empirical estimates.

Now let us calculate the world population in 1900. It is clear that in order to do this we should simply divide 215000 million by 127; this gives 1693 million, which turns out to be precisely within the range of the extant empirical estimates (1600–1710 million).⁵

Let us do the same operation for the year 1800: $2027 - 1800 = 227$; $215000 \div 227 = 947.1$ (million). According to empirical estimates, the world population for 1800 indeed was between 900 and 980 million.⁶ Let us repeat the operation for 1700: $2027 - 1700 = 327$; $215000 \div 327 = 657$ (million). Once again, we find ourselves within the margins of available empirical estimates (600–679 million).⁷ Let us repeat the algorithm once more, for the year 1400: $2027 - 1400 = 627$; $215000 \div 627 = 343$ (million). Yet again, we see that the result falls within the error margins of available world population estimates for this date.⁸ The overall correlation between the curve generated by von Foerster's

⁴ Note that the value of coefficient k (equivalent to parameter λ in equation (1)) used by us is a bit different from the one used by von Foerster.

⁵ Thomlinson 1975; Durand 1977; McEvedy and Jones 1978; Biraben 1980; Haub 1995; Modelski 2003; UN Population Division 2006; U.S. Bureau of the Census 2006.

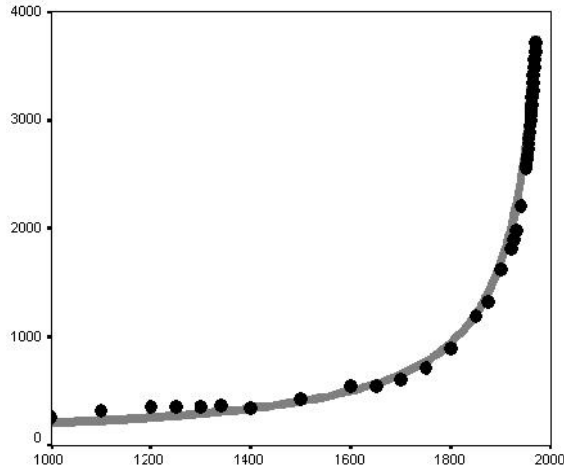
⁶ Thomlinson 1975; McEvedy and Jones 1978; Biraben 1980; Modelski 2003; UN Population Division 2006; U.S. Bureau of the Census 2006.

⁷ Thomlinson 1975; McEvedy and Jones 1978; Biraben 1980; Maddison 2001; Modelski 2003; U.S. Bureau of the Census 2006.

⁸ 350 million (McEvedy and Jones 1978), 374 million (Biraben 1980).

equation and the most detailed series of empirical estimates looks as follows (see Diagram 0.3):

Diagram 0.3. Correlation between Empirical Estimates of World Population (in millions, 1000 – 1970) and the Curve Generated by von Foerster's Equation



NOTE: black markers correspond to empirical estimates of the world population by McEvedy and Jones (1978) for 1000–1950 and the U.S. Bureau of the Census (2006) for 1950–1970. The grey curve has been generated by von Foerster's equation (0.4).

The formal characteristics are as follows: $R = 0.998$; $R^2 = 0.996$; $p = 9.4 \times 10^{-17} - 1 \times 10^{-16}$. For readers unfamiliar with mathematical statistics: R^2 can be regarded as a measure of the fit between the dynamics generated by a mathematical model and the empirically observed situation, and can be interpreted as the proportion of the variation accounted for by the respective equation. Note that 0.996 also can be expressed as 99.6%.⁹ Thus, von Foerster's equation accounts for an astonishing 99.6% of all the macrovariation in world population, from 1000 CE through 1970, as estimated by McEvedy and Jones (1978) and the U.S. Bureau of the Census (2006).¹⁰

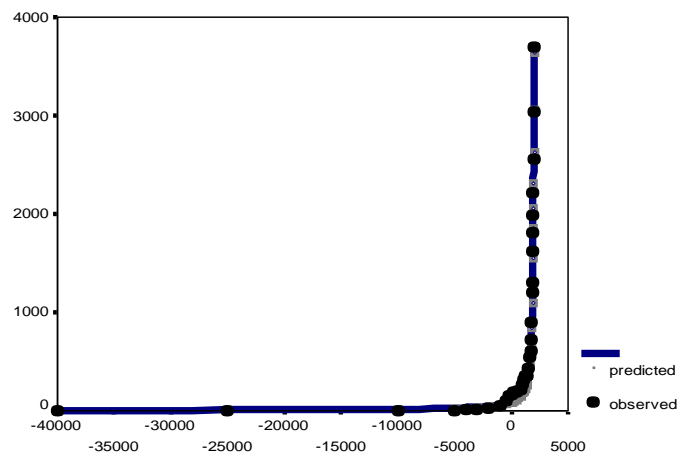
⁹ The second characteristic (p , standing for "probability") is a measure of the correlation's statistical significance. A bit counterintuitively, the lower the value of p , the higher the statistical significance of the respective correlation. This is because p indicates the probability that the observed correlation could be accounted solely by chance. Thus, $p = 0.99$ indicates an extremely low statistical significance, as it means that there are 99 chances out of 100 that the observed correlation is the result of a coincidence, and, thus, we can be quite confident that there is no systematic relationship (at least, of the kind that we study) between the two respective variables. On the other hand, $p = 1 \times 10^{-16}$ indicates an extremely high statistical significance for the correlation, as it means that there is only one chance out of 10000000000000000 that the observed correlation is the result of pure coincidence (in fact, a correlation is usually considered as statistically significant with $p < 0.05$).

¹⁰ In fact, with slightly different parameters ($\lambda = 164890.45$; $t_0 = 2014$) the fit (R^2) between the dynamics generated by von Foerster's equation and the macrovariation of world population for CE 1000 – 1970 as estimated by McEvedy and Jones (1978) and the U.S. Bureau of the Census

Note also that the empirical estimates of world population find themselves aligned in an extremely neat way along the hyperbolic curve, which convincingly justifies the designation of the pre-1970s world population growth pattern as "hyperbolic".

Von Foerster and his colleagues detected the hyperbolic pattern of world population growth for 1 CE –1958 CE; later it was shown that this pattern continued for a few years after 1958,¹¹ and also that it can be traced for many millennia BCE (Kapitza 1992, 1999; Kremer 1993).¹² Indeed, the McEvedy and Jones (1978) estimates for world population for the period 5000–500 BCE are described rather accurately by a hyperbolic equation ($R^2 = 0.996$); and this fit remains rather high for 40000 – 200 BCE ($R^2 = 0.990$) (see below Appendix 2). The overall shape of the world's population dynamics in 40000 BCE – 1970 CE also follows the hyperbolic pattern quite well (see Diagram 0.4):

Diagram 0.4. World Population Dynamics, 40000 BCE – 1970 CE (in millions): the fit between predictions of a hyperbolic model and the observed data



NOTE: $R = 0.998$, $R^2 = 0.996$, $p \ll 0.0001$. Black markers correspond to empirical estimates of the world population by McEvedy and Jones (1978) and Kremer (1993) for 1000–1950, as well as

(2006) reaches 0.9992 (99.92%), whereas for 500 BCE – 1970 CE this fit increases to 0.9993 (99.93%) (with the following parameters: $\dots = 171042.78$; $t_0 = 2016$).

¹¹ Note that after the 1960s, world population deviated from the hyperbolic pattern more and more; at present it definitely is no longer hyperbolic (see, *e.g.*, Korotayev, Malkov, and Khaltourina 2006a: Chapter 1).

¹² In fact, Kremer asserts the presence of this pattern since 1 million BCE; Kapitza, since 4 million BCE! We, however, are not prepared to accept these claims, because it is far from clear even who constituted the "world population" in, say, 1 million BCE, let alone how their number could have been empirically estimated.

the U.S. Bureau of the Census (2006) data for 1950–1970. The solid line has been generated by the following version of von Foerster's equation:

$$N_t = \frac{189648.7}{2022 - t}.$$

A usual objection (*e.g.*, Shishkov 2005) against the statement that the overall pattern of world population growth until the 1970s was hyperbolic is as follows. Since we simply do not know the exact population of the world for most of human history (and especially, before CE), we do not have enough information to detect the general shape of the world population dynamics through most of human history. Thus, there are insufficient grounds to accept the statement that the overall shape of the world population dynamics in 40000 BCE – 1970 CE was hyperbolic.

At first glance this objection looks very convincing. For example, for 1 BCE the world population estimates range from 170 million (McEvedy and Jones 1978) to 330 million (Durand 1977), whereas for 10000 BCE the estimate range becomes even more dramatic: 1–10 million (Thomlinson 1975). Indeed, it seems evident that with such uncertain empirical data, we are simply unable to identify the long-term trend of world population macrodynamics.

However, notwithstanding the apparent persuasiveness of this objection, we cannot accept it. Let us demonstrate why.

Let us start with 10000 CE. As was mentioned above, we have only a rather vague idea about how many people lived on the Earth that time. However, we can be reasonably confident that it was more than 1 million, and less than 10 million. Note that this is not even a guesstimate. Indeed, we know which parts of the world were populated by that time (most of it, in fact), what kind of subsistence economies were practiced¹³ (see, *e.g.*, Peregrine and Ember 2001), and what the maximum number of people 100 square kilometers could support with any of these subsistence economies (see, *e.g.*, Korotayev 1991). Thus, we know that with foraging technologies practiced by human populations in 10000 BCE, the Earth could not have supported more than 10 million people (and the actual world population is very likely to have been substantially smaller). Regarding world population in 40000 BCE, we can be sure only that it was somewhat smaller than in 10000 BCE. We do not know what exactly the difference was, but as we shall see below, this is not important for us in the context of this discussion.

The available estimates of world population between 10000 BCE and 1 CE can, of course, be regarded as educated guesstimates. However, in 2 CE the situation changes substantially, because this is the year of the "earliest preserved census in the world" (Bielenstein 1987: 14). Note also that this census was performed in China, one of the countries that is most important for us in this con-

¹³ Note that at that time these economies were exclusively foraging (though quite intensive in a few areas of the world [see, *e.g.*, Grinin 2003b]).

text. This census recorded 59 million taxable inhabitants of China (*e.g.*, Bielenstein 1947: 126, 1986: 240; Durand 1960: 216; Loewe 1986: 206), or 57.671 million according to a later re-evaluation by Bielenstein (1987: 14).¹⁴ Up to the 18th century the Chinese counts tended to underestimate the population, since before this they were not real census, but rather registrations for taxation purposes; in any country a large number of people would do their best to escape such a registration in order to avoid paying taxes, and it is quite clear that some part of the Chinese population normally succeeded in this (see, *e.g.*, Durand 1960). Hence, at least we can be confident that in 2 CE the world population was no less than 57.671 million. It is also quite clear that the world population was substantially more than that. For this time we also have data from a census of the Roman citizenry (for 14 CE), which, together with information on Roman social structure and data from narrative and archaeological sources, makes it possible to identify with a rather high degree of confidence the order of magnitude of the population of the Roman Empire (with available estimates in the range of 45–80 million [Durand 1977: 274]). Textual sources and archaeological data also make it possible to identify the order of magnitude of the population of the Parthian Empire (10–20 million), and of India (50–100 million) (Durand 1977). Data on the population for other regions warrant less confidence, but it is still quite clear that their total population was much smaller than that of the four above-mentioned regions (which in 2 CE comprised most of the world population). Archaeological evidence suggests that population density for the rest of the world would have been considerably lower than in the "Four Regions" themselves. In general, then, we can be quite sure that the world population in 2 CE could scarcely have been less than 150 million; it is very unlikely that it was more than 350 million.

Let us move now to 1800 CE. For this time we have much better population data than ever before for most of Europe, the United States, China¹⁵, Egypt¹⁶, India, Japan, and so on (Durand 1977). Hence, for this year we can be quite confident that world population could scarcely have been less than 850 million and more than 1 billion. The situation with population statistics further improves by 1900¹⁷ for which time there is not much doubt that world population this year was within the range of 1600–1750 million. Finally, by 1960 population statistics had improved dramatically, and we can be quite confident that world population then was within the range of 2900–3100 million.

¹⁴ Or 57.671 million according to a later re-evaluation by Bielenstein (1987: 14).

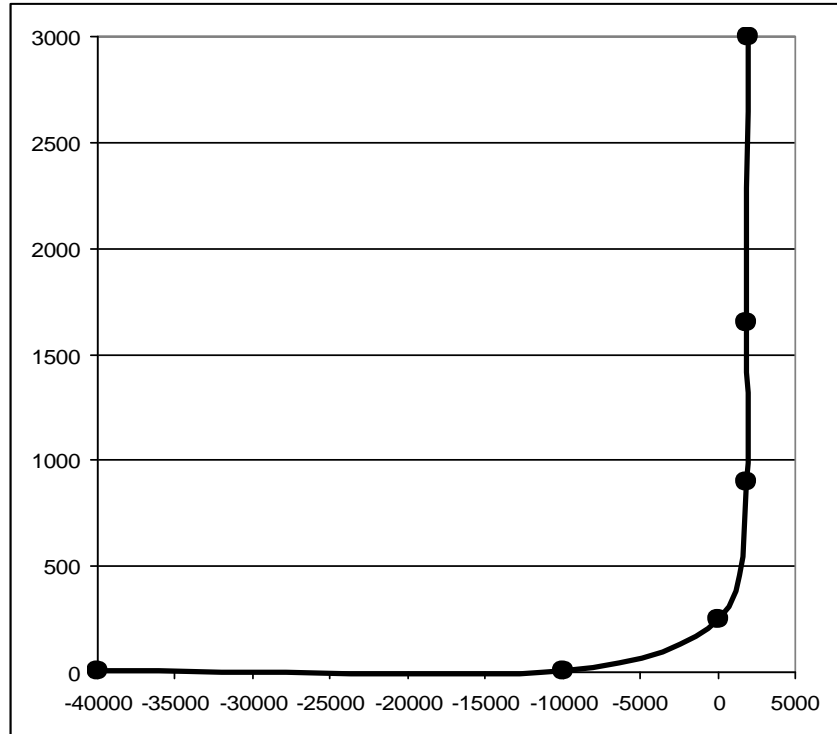
¹⁵ Due to the separation of the census registration from the tax assessment conducted in the first half of the 18th century, the Chinese population in 1800 had no substantive reason for avoiding the census registration. Therefore, the Chinese census data for this time are particularly reliable (*e.g.*, Durand 1960: 238; see also Chapter 2 of this book).

¹⁶ Due to the first scientific estimation of the Egyptian population performed by the members of the scientific mission that accompanied Napoleon to Egypt (Jomard 1818).

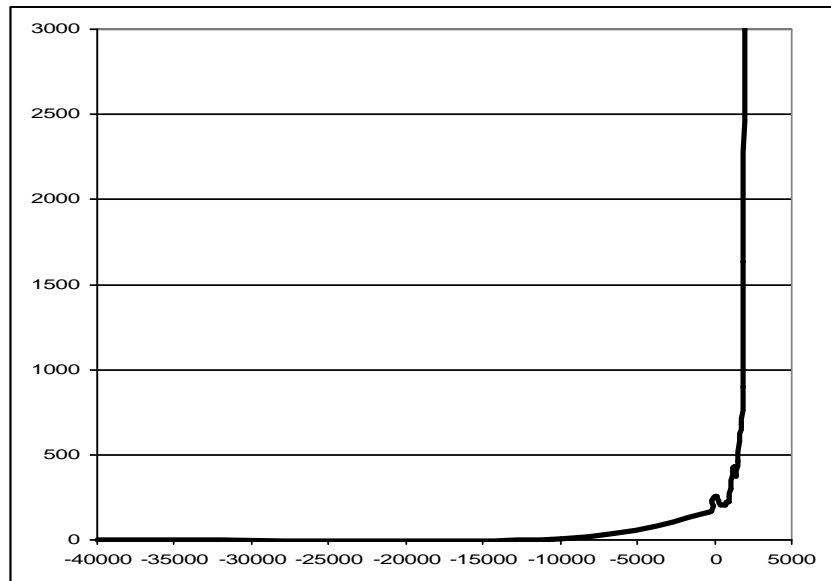
¹⁷ With a notable exception of China (Durand 1960; see also below Chapter 2).

Now let us plot the mid points of the above mentioned estimate ranges and connect the respective points. We will get the following picture (see Diagram 0.5):

Diagram 0.5



As we see the resulting pattern of world population dynamics has an unmistakably hyperbolic shape. Now you can experiment and move any points within the estimate ranges as much as you like. You will see that the overall hyperbolic shape of the long-term world population dynamics will remain intact. What is more, you can fill the space between the points with any estimates you find. You will see that the overall shape of the world population dynamics will always remain distinctly hyperbolic. Replace, for example, the estimates of McEvedy and Jones (1978) used by us earlier for Diagram 0.4 in the range between 10000 BCE and 1900 CE with the ones of Biraben (1980) (note that generally Biraben's estimates are situated in the opposite side of the estimate range in relation to the ones of McEvedy and Jones). You will get the following picture (see Diagram 0.6):

Diagram 0.6

As we see, the overall shape of the world population dynamics remains unmistakably hyperbolic.

So what is the explanation for this apparent paradox? Why, though world population estimates are evidently infirm for most of human history, can we be sure that long-term world population dynamics pattern was hyperbolic?

The answer is simple, for in the period in question the world population grew by orders of magnitude. It is true that for most part of human history we cannot be at all confident of the exact value within a given order of magnitude. But with respect to any time-point within any period in question, we can be already perfectly confident about the order of magnitude of the world population. Hence, it is clear that whatever discoveries are made in the future, whatever re-evaluations are performed, the probability that they will show that the overall world population growth pattern in 40000 BCE – 1970 CE was not hyperbolic (but, say, exponential or lineal) is very close to zero indeed.

Note that if von Foerster, Mora, and Amiot also had at their disposal, in addition to world population data, data on the world GDP dynamics for 1–1973 (published, however, only in 2001 by Maddison [Maddison 2001]), they could have made another striking "prediction" – that on Saturday, 23 July, A.D. 2005 an "economic doomsday" would take place; that is, on that day the world GDP would become infinite if the economic growth trend observed in 1–1973 CE

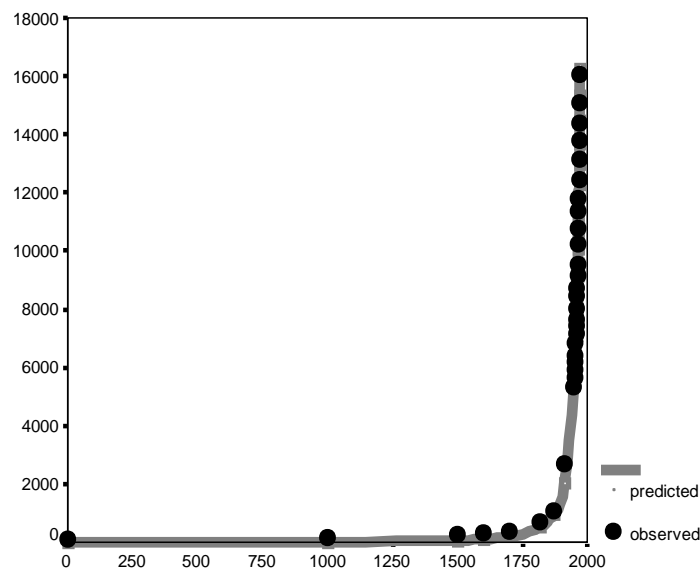
continued. They also would have found that in 1–1973 CE the world GDP growth followed a quadratic-hyperbolic rather than simple hyperbolic pattern.

Indeed, Maddison’s estimates of the world GDP dynamics for 1–1973 CE are almost perfectly approximated by the following equation:

$$G_t = \frac{C}{(t_0 - t)^2}, \tag{0.5}$$

where G_t is the world GDP (in billions of 1990 international dollars, in purchasing power parity [PPP]) in year t , $C = 17355487.3$ and $t_0 = 2005.56$ (see Diagram 0.7):

Diagram 0.7. World GDP Dynamics, 1–1973 CE (in billions of 1990 international dollars, PPP): the fit between predictions of a quadratic-hyperbolic model and the observed data



NOTE: $R = .9993$, $R^2 = .9986$, $p \ll .0001$. The black markers correspond to Maddison’s (2001) estimates (Maddison’s estimates of the world per capita GDP for 1000 CE has been corrected on the basis of Meliantsev [1996, 2003, 2004a, 2004b]). The grey solid line has been generated by the following equation:

$$G = \frac{17749573.1}{(2006 - t)^2}.$$

Actually, as was mentioned above, the best fit is achieved with $k = 17355487.3$ and $t_0 = 2005.56$ (which gives just the "doomsday Saturday, 23 July, 2005"), but we have decided to keep hereafter to integer numbered years.

The only difference between the simple and quadratic hyperbolas is that the simple hyperbola is described mathematically with equation (0.2):

$$y = \frac{k}{x}, \quad (0.2)$$

whereas the quadratic hyperbolic equation has x^2 instead of just x :

$$y = \frac{k}{x^2}. \quad (0.6)$$

Of course, this equation can also be written as follows:

$$y = \frac{k}{(x_0 - x)^2}. \quad (0.7)$$

It is this equation that was used above to describe the world economic dynamics between 1 and 1973 CE. The algorithm for calculating the world GDP still remains very simple. *E.g.*, to calculate the world GDP in 1905 (in billions of 1990 international dollars, PPP), one should first subtract 1905 from 2005, but then to divide k (17355487.3) not by the resultant difference (100), but by its square ($100^2 = 10000$).

Those readers who are not familiar with mathematical models of population hyperbolic growth should have a lot of questions at this point.¹⁸ How could the long-term macrodynamics of the most complex social system be described so accurately with such simple equations? Why do these equations look so strange? Why, indeed, can we estimate the world population in year x so accurately just by subtracting x from the "Doomsday" year and dividing some constant with the resultant difference? And why, if we want to know the world GDP in this year, should we square the difference prior to dividing? Why was the hyperbolic growth of the world population accompanied by the quadratic hyperbolic growth of the world GDP? Is this a coincidence? Or are the hyperbolic growth of the world population and the quadratic hyperbolic growth of the world GDP just two sides of one coin, two logically connected aspects of the same process?

In the first part of our *Introduction to Social Macrodynamics* we have tried to provide answers to this question and these answers are summarized below.

¹⁸ Whereas the answers to the questions regarding the quadratic hyperbolic growth of the world GDP might not have been quite clear even for those readers who know the hyperbolic demographic models.

However, before starting this we would like to state that our experience shows that most readers who are not familiar with mathematics stop reading books (at least our books) as soon as they come across the words – "differential equation". Thus, we have to ask such readers not to get scared with the presence of these words in the next passage and to move further. You will see that it is not as difficult to understand differential equations (or, at least, some of those equations), as one might think.

To start with, the von Foerster equation, $N_t = \frac{C}{t_0 - t}$, is just the solution for the following differential equation (see, *e.g.*, Korotayev, Malkov, and Khal-tourina 2006a: 119–20):

$$\frac{dN}{dt} = \frac{N^2}{C}. \quad (0.8)$$

This equation can be also written as:

$$\frac{dN}{dt} = aN^2, \quad (0.9)$$

where $a = \frac{1}{C}$.

What is the meaning of this mathematical expression, $\frac{dN}{dt} = aN^2$? In our context dN/dt denotes the absolute population growth rate at some moment of time. Hence, this equation states that the absolute population growth rate at any moment of time should be proportional to the square of population at this moment.

Note that by dividing both parts of equation (0.9) with N we will get the following:

$$\frac{dN}{dt} : N = aN, \quad (0.10)$$

Further, note that $\frac{dN}{dt} : N$ is just a designation of the relative population growth rate. Indeed, as we remember, dN/dt is the absolute population growth rate at a certain moment of time. Imagine that at this moment the population (N) is 100 million and the absolute population growth rate (dN/dt) is 1 million a year. If we divide now ($dN/dt = 1$ million) by ($N = 100$ million) we will get 0.01, or 1%; which would mean that the relative population growth rate at this moment is 1% a year.

If we denote relative population growth rate as r_N , we will get a particularly simple version of the hyperbolic equation:

$$r_N = aN. \quad (0.10)$$

Thus, with hyperbolic growth the relative population growth rate (r_N) is linearly proportional to the population size (N). Note that this significantly demystifies the problem of the world population hyperbolic growth. Now to explain this hyperbolic growth we should just explain why for many millennia the world population's absolute growth rate tended to be proportional to the square of the population.

We believe that the most significant progress towards the development of a compact mathematical model providing a convincing answer to this question has been achieved by Michael Kremer (1993), whose model will be summarized next.

Kremer's model is based on the following assumptions:

1) First of all he makes "the Malthusian (1978) assumption that population is limited by the available technology, so that the growth rate of population is proportional to the growth rate of technology" (Kremer 1993: 681–2).¹⁹ This statement looks quite convincing. Indeed, throughout most of human history the world population was limited by the technologically determined ceiling of the carrying capacity of land. As was mentioned above, with foraging subsistence technologies the Earth could not support more than 10 million people, because the amount of naturally available useful biomass on this planet is limited, and the world population could only grow over this limit when the people started to apply various means to artificially increase the amount of available biomass, that is with the transition from foraging to food production. However, the extensive agriculture also can only support a limited number of people, and further growth of the world population only became possible with the intensification of agriculture and other technological improvements.

This assumption is modeled by Kremer in the following way. Kremer assumes that overall output produced by the world economy equals

$$G = rTN^r, \quad (0.11)$$

where G is output, T is the level of technology, N is population, $0 < r < 1$ and r are parameters.²⁰ With constant T (that is, without any technological growth)

¹⁹ In addition to this, the absolute growth rate is proportional to the population itself – with a given relative growth rate a larger population will increase more in absolute numbers than a smaller one.

²⁰ Kremer uses the following symbols to denote respective variables: Y – output, p – population, A – the level of technology, *etc.*; while describing Kremer's models we will employ the symbols (closer to the Kapitza's [1992, 1999]) used in our model, naturally without distorting the sense of Kremer's equations.

this equation generates Malthusian dynamics. For example, let us assume that $\alpha = 0.5$, and that T is constant. Let us recollect that $N^{0.5}$ is just \sqrt{N} . Thus, a four time expansion of the population will lead to a twofold increase in output (as $\sqrt{4} = 2$). In fact, here Kremer models Ricardo's law of diminishing returns to labor (1817), which in the absence of technological growth produces just Malthusian dynamics. Indeed, if the population grows 4 times, and the output grows only twice, this will naturally lead to a twofold decrease of per capita output. How could this affect population dynamics?

Kremer assumes that "population increases above some steady state equilibrium level of per capita income, m , and decreases below it" (Kremer 1993: 685). Hence, with the decline of per capita income, the population growth will slow down and will become close to zero when the per capita income approaches m . Note that such a dynamics was actually rather typical for agrarian societies, and its mechanisms are known very well – indeed, if per capita incomes decline closely to m , it means the decline of nutrition and health status of most population, which will lead to an increase in mortality and a slow down of population growth (see, *e.g.*, Malthus 1798 [1798]; Postan 1950, 1972; Abel 1974, 1980; Cameron 1989; Artzrouni and Komlos 1985; Komlos and Nefedov 2002; Turchin 2003; Nefedov 2004 and Chapters 1–3 below). Thus, with constant technology, population will not be able to exceed the level at which per capita income ($g = G/N$) becomes equal to m . This implies that for any given level of technological development (T) there is "a unique level of population, \bar{n} ," that cannot be exceeded with the given level of technology (Kremer 1993: 685). Note that \bar{n} can be also interpreted as the Earth carrying capacity, that is, the maximum number of people that the Earth can support with the given level of technology.

However, as is well known, the technological level is not a constant, but a variable. And in order to describe its dynamics Kremer employs his second basic assumption:

2) "High population spurs technological change because it increases the number of potential inventors...²¹ In a larger population there will be proportionally more people lucky or smart enough to come up with new ideas" (Kremer 1993: 685), thus, "the growth rate of technology is proportional to total population".²² In fact, here Kremer uses the main assumption of the Endogenous Technological Growth theory (Kuznets 1960; Grossman and Helpman 1991; Aghion and Howitt 1992, 1998; Simon 1977, 1981, 2000; Komlos and Nefedov

²¹ "This implication flows naturally from the nonrivalry of technology... The cost of inventing a new technology is independent of the number of people who use it. Thus, holding constant the share of resources devoted to research, an increase in population leads to an increase in technological change" (Kremer 1993: 681).

²² Note that "the growth rate of technology" means here the relative growth rate (*i.e.*, the level to which technology will grow in a given unit of time in proportion to the level observed at the beginning of this period).

2002; Jones 1995, 2003, 2005 *etc.*). As this supposition, to our knowledge, was first proposed by Simon Kuznets (1960), we shall denote the corresponding type of dynamics as "Kuznetsian",²³ while the systems in which the "Kuznetsian" population-technological dynamics is combined with the "Malthusian" demographic one will be denoted as "Malthusian-Kuznetsian". In general, we find this assumption rather plausible – in fact, it is quite probable that, other things being equal, within a given period of time, one billion people will make approximately one thousand times more inventions than one million people.

This assumption is expressed by Kremer mathematically in the following way:

$$\frac{dT}{dt} = bNT \quad (0.12)$$

Actually, this equation says just that the absolute technological growth rate at a given moment of time is proportional to the technological level observed at this moment (the wider is the technological base, the more inventions could be made on its basis), and, on the other hand, it is proportional to the population (the larger the population, the higher the number of potential inventors).²⁴

In his basic model Kremer assumes "that population adjusts instantaneously to \bar{n} " (1993: 685); he further combines technology and population determination equations and demonstrates that their interaction produces just the hyperbolic population growth (Kremer 1993: 685–6; see also Podlazov 2000, 2001, 2002, 2004; Tsirel 2004; Korotayev, Malkov, and Khaltourina 2006a: 21–36).

Kremer's model provides a rather convincing explanation of *why* throughout most of human history the world population followed the hyperbolic pattern with the absolute population growth rate tending to be proportional to N^2 . For example, why will the growth of population from, say, 10 million to 100 million, result in the growth of dN/dt 100 times? Kremer's model explains this rather convincingly (though Kremer himself does not appear to have spelled this out in a sufficiently clear way). The point is that the growth of world population from 10 to 100 million implies that human technology also grew approximately 10 times (given that it will have proven, after all, to be able to support a ten times larger population). On the other hand, the growth of a population 10 times also implies a 10-fold growth of the number of potential inventors, and, hence, a 10-fold increase in the relative technological growth rate. Hence, the absolute technological growth rate will grow $10 \times 10 = 100$ times (as, in accordance with equation (0.12), an order of magnitude higher number of people having at their disposal an order of magnitude wider technological basis would tend to make

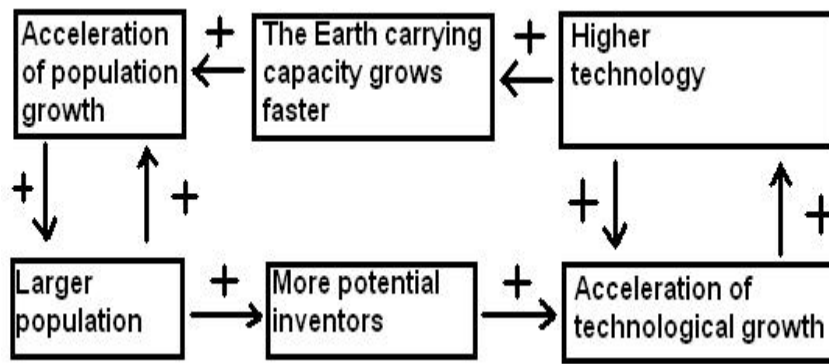
²³ In Economic Anthropology it is usually denoted as "Boserupian" (see, *e.g.*, Boserup 1965; Lee 1986).

²⁴ Kremer did not test this hypothesis empirically in a direct way. Note, however, that our own empirical test of this hypothesis has supported it (see below Appendix 1).

two orders of magnitude more inventions). And as N tends to the technologically determined carrying capacity ceiling, we have good reason to expect that dN/dt will also grow just by about 100 times.

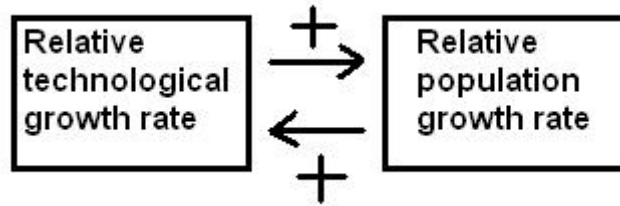
In fact, Kremer's model suggests that the hyperbolic pattern of the world's population growth could be accounted for by the nonlinear second order positive feedback mechanism that was shown long ago to generate just the hyperbolic growth, known also as the "blow-up regime" (see, *e.g.*, Kurdjumov 1999; Knjazeva and Kurdjumov 2005). In our case this nonlinear second order positive feedback looks as follows: the more people – the more potential inventors – the faster technological growth – the faster growth of the Earth's carrying capacity – the faster population growth – with more people you also have more potential inventors – hence, faster technological growth, and so on (see Diagram 0.8):

Diagram 0.8. Block Scheme of the Nonlinear Second Order Positive Feedback between Technological Development and Demographic Growth (version 1)



In fact, this positive feedback can be graphed even more succinctly (see Diagram 0.9a):

Diagram 0.9a. Block Scheme of the Nonlinear Second Order Positive Feedback between Technological Development and Demographic Growth (version 2)



Note that the relationship between technological development and demographic growth cannot be analyzed through any simple cause-and-effect model, as we observe a true dynamic relationship between these two processes – each of them is both the cause and the effect of the other.

It is remarkable that Kremer's model suggests ways to answer one of the main objections raised against the hyperbolic models of the world's population growth. Indeed, at present the mathematical models of world population growth as hyperbolic have not been accepted by the academic social science community [The title of the most recent article by a social scientist discussing Kapitza's model, "Demographic Adventures of a Physicist" (Shishkov 2005), is rather telling in this respect]. We believe that there are substantial reasons for such a position, and that the authors of the respective models are as much to blame for this rejection as are social scientists.

Indeed, all these models are based on an assumption that world population can be treated as having been an integrated system for many centuries, if not millennia, before 1492. Already in 1960, von Foerster, Mora, and Amiot spelled out this assumption in a rather explicit way:

"However, what may be true for elements which, because of lack of adequate communication among each other, have to resort to a competitive, (almost) zero-sum multiperson game may be false for elements that possess a system of communication which enables them to form coalitions until all elements are so strongly linked that the population as a whole can be considered from a game-theoretical point of view as a single person playing a two-person game with nature as its opponent" (von Foerster, Mora, and Amiot 1960: 1292).

However, did, *e.g.*, in 1–1500 CE, the inhabitants of, say, Central Asia, Tasmania, Hawaii, Terra del Fuego, the Kalahari *etc.* (that is, just the world population) really have "adequate communication" to make "all elements... so strongly linked that the population as a whole can be considered from a game-theoretical point of view as a single person playing a two-person game with nature as its opponent"? For any historically minded social scientist the answer to this question is perfectly clear and, of course, it is squarely negative. Against

this background it is hardly surprising that those social scientists who have happened to come across hyperbolic models for world population growth have tended to treat them merely as "demographic adventures of physicists" (note that indeed, nine out of eleven currently known authors of such models are physicists); none of the respective authors (von Foerster, Mora, and Amiot 1960; von Hoerner 1975; Kapitza 1992, 1999; Kremer 1993; Cohen 1995; Podlazov 2000, 2001, 2002, 2004; Johansen and Sornette 2001; Tsirel 2004), after all, has provided any convincing answer to the question above.

However, it is not so difficult to provide such an answer.

The hyperbolic trend observed for the world population growth after 10000 BCE does appear to be primarily a product of the growth of quite a real system, a system that seems to have originated in West Asia around that time in direct connection with the Neolithic Revolution. With Andre Gunder Frank (1990, 1993; Frank and Gills 1994), we denote this system as "the World System" (see also, *e.g.*, Modelski 2000, 2003; Devezas and Modelski 2003). The presence of the hyperbolic trend itself indicates that the major part of the entity in question had some systemic unity, and the evidence for this unity is readily available. Indeed, we have evidence for the systematic spread of major innovations (domesticated cereals, cattle, sheep, goats, horses, plow, wheel, copper, bronze, and later iron technology, and so on) throughout the whole North African – Eurasian Oikumene for a few millennia BCE (see, *e.g.*, Chubarov 1991, or Diamond 1999 for a synthesis of such evidence). As a result, the evolution of societies of this part of the world already at this time cannot be regarded as truly independent. By the end of the 1st millennium BCE we observe a belt of cultures, stretching from the Atlantic to the Pacific, with an astonishingly similar level of cultural complexity characterized by agricultural production of wheat and other specific cereals, the breeding of cattle, sheep, and goats; use of the plow, iron metallurgy, and wheeled transport; development of professional armies and cavalries deploying rather similar weapons; elaborate bureaucracies, and Axial Age ideologies, and so on – this list could be extended for pages). A few millennia before, we would find another belt of societies strikingly similar in level and character of cultural complexity, stretching from the Balkans up to the Indus Valley outskirts (Peregrine and Ember 2001: vols. 4 and 8; Peregrine 2003). Note that in both cases, the respective entities included the major part of the contemporary world's population (see, *e.g.* McEvedy and Jones 1978; Durand 1977 *etc.*). We would interpret this as a tangible result of the World System's functioning. The alternative explanations would involve a sort of miraculous scenario – that these cultures with strikingly similar levels and character of complexity somehow developed independently of one another in a very large but continuous zone, while for some reason nothing comparable to them appeared elsewhere in the other parts of the world, which were not parts of the World System. We find such an alternative explanation highly implausible.

Thus, we would tend to treat the world population's hyperbolic growth pattern as reflecting the growth of quite a real entity, the World System.

A few other points seem to be relevant here. Of course there would be no grounds for speaking about a World System stretching from the Atlantic to the Pacific, even at the beginning of the 1st millennium CE, if we applied the "bulk-good" criterion suggested by Wallerstein (1974, 1987, 2004), as there was no movement of bulk goods at all between, say, China and Europe at this time (as we have no reason to disagree with Wallerstein in his classification of the 1st century Chinese silk reaching Europe as a luxury rather than a bulk good). However, the 1st century CE (and even the 1st millennium BCE) World System definitely qualifies as such if we apply the "softer" information-network criterion suggested by Chase-Dunn and Hall (1997). Note that at our level of analysis the presence of an information network covering the whole World System is a perfectly sufficient condition, which makes it possible to consider this system as a single evolving entity. Yes, in the 1st millennium BCE any bulk goods could hardly penetrate from the Pacific coast of Eurasia to its Atlantic coast. However, the World System had reached by that time such a level of integration that iron metallurgy could spread through the whole of the World System within a few centuries.

Yes, in the millennia preceding the European colonization of Tasmania its population dynamics – oscillating around the 4000 level (*e.g.*, Diamond 1999) – were not influenced by World System population dynamics and did not influence it at all. However, such facts just suggest that since the 10th millennium BCE the dynamics of the world population reflect very closely just the dynamics of the World System population.

On the basis of Kremer's model we (Korotayev, Malkov, and Khaltourina 2006a: 34–66) have developed a mathematical model that describes not only the hyperbolic world population growth, but also the macrodynamics of the world GDP production up to 1973:

$$G = k_1 TN^r, \quad (0.11)$$

$$\frac{dN}{dt} = k_2 SN, \quad (0.13)$$

$$\frac{dT}{dt} = k_3 NT, \quad (0.12)$$

where G is the world GDP, T is the World System technological level, N is population, and S is the surplus produced, per person, over the amount (m) minimally necessary to reproduce the population with a zero growth rate in a Malthusian system (thus, $S = g - m$, where g denotes per capita GDP); k_1 , k_2 , k_3 , and $(0 < r < 1)$ are parameters.

We have also shown (Korotayev, Malkov, and Khaltourina 2006a: 34–66) that this model can be further simplified to the following form:

$$\frac{dN}{dt} = aSN , \quad (0.13)$$

$$\frac{dS}{dt} = bNS , \quad (0.14)$$

while the world GDP (G) can be calculated using the following equation:

$$G = mN + SN . \quad (0.15)$$

Note that the mathematical analysis of the basic model (0.11)-(0.13)-(0.12) suggests that during the "Malthusian-Kuznetsian" macroperiod of human history (that is, up to the 1960s) the amount of S (per capita surplus produced at the given level of World System development) should be proportional, in the long run, to the World System's population: $S = kN$. Our statistical analysis of available empirical data has confirmed this theoretical proportionality (Korotayev, Malkov, and Khaltourina 2006a: 49–50). Thus, in the right-hand side of equation (0.13) S can be replaced with kN , and as a result we arrive at the following equation:

$$\frac{dN}{dt} = kaN^2 \quad (0.9)^{25}$$

As we remember, the solution of this type of differential equations is

$$N_t = \frac{C}{(t_0 - t)} , \quad (0.1)$$

and this produces simply a hyperbolic curve.

As, according to our model, S can be approximated as kN , its long-term dynamics can be approximated with the following equation:

$$S = \frac{kC}{t_0 - t} . \quad (0.16)$$

Thus, the long-term dynamics of the most dynamic component of the world GDP, SN , "the world surplus product", can be approximated as follows:

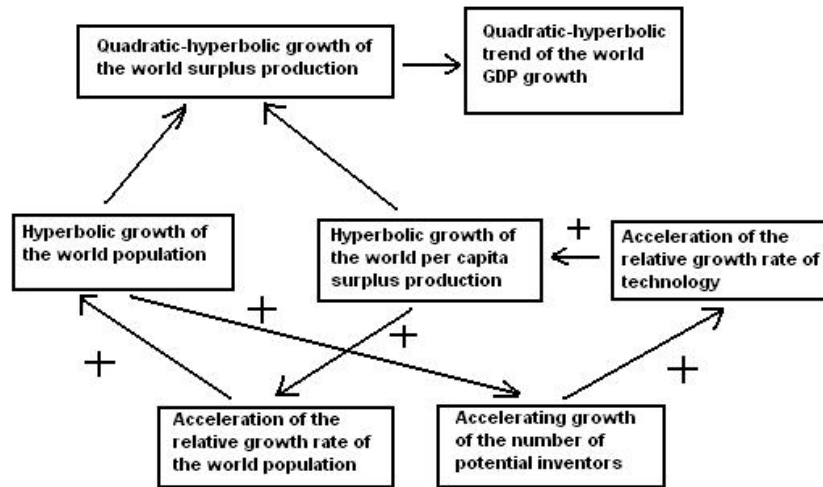
$$SN = \frac{kC^2}{(t_0 - t)^2} . \quad (0.17)$$

²⁵ Thus we arrive, on a theoretical basis, at the differential equation discovered empirically by von Hoerner (1975) and Kapitza (1992, 1999).

Of course, this suggests that the long-term world GDP dynamics up to the early 1970s must be approximated better by a quadratic hyperbola than by a simple one; and, as we could see above (see Diagram 0.7), this approximation works very effectively indeed.

Thus, up to the 1970s the hyperbolic growth of the world population was accompanied by the quadratic-hyperbolic growth of the world GDP, just as is suggested by our model. Note that the hyperbolic growth of the world population and the quadratic hyperbolic growth of the world GDP are very tightly connected processes, actually two sides of the same coin, two dimensions of one process propelled by the nonlinear second order positive feedback loops between the technological development and demographic growth (see Diagram 0.9b):

Diagram 0.9b. Block Scheme of the Nonlinear Second Order Positive Feedback between Technological Development and Demographic Growth (version 3)



We have also demonstrated (Korotayev, Malkov, and Khaltourina 2006a: 67–80) that the World System population's literacy (l) dynamics are rather accurately described by the following differential equation:

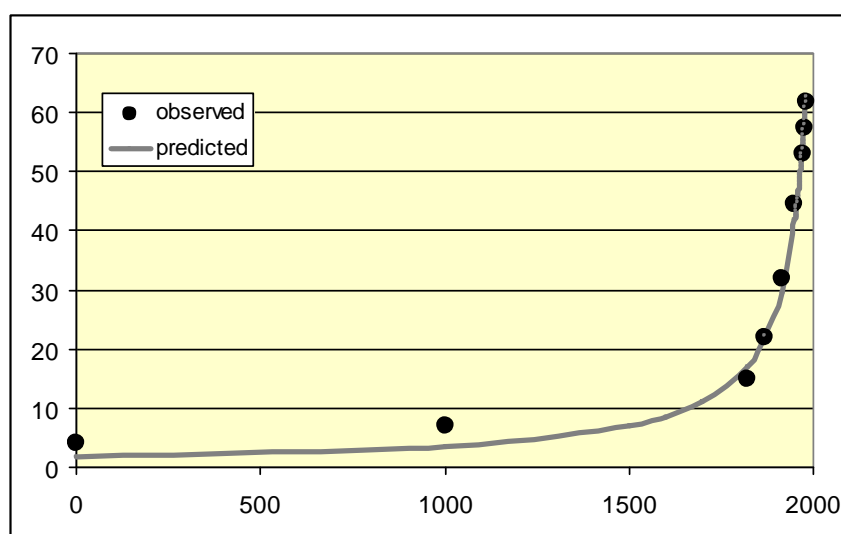
$$\frac{dl}{dt} = aSl(1-l), \quad (0.18)$$

where l is the proportion of the population that is literate, S is per capita surplus, and a is a constant. In fact, this is a version of the autocatalytic model. It has the following sense: the literacy growth is proportional to the fraction of the population that is literate, l (potential teachers), to the fraction of the population that

is illiterate, $(1 - l)$ (potential pupils), and to the amount of per capita surplus S , since it can be used to support educational programs (in addition to this, S reflects the technological level T that implies, among other things, the level of development of educational technologies). Note that, from a mathematical point of view, equation (0.18) can be regarded as logistic where saturation is reached at literacy level $l = 1$, and S is responsible for the speed with which this level is being approached.

It is important to stress that with low values of l (which would correspond to most of human history, with recent decades being the exception), the rate of increase in world literacy generated by this model (against the background of hyperbolic growth of S) can be approximated rather accurately as hyperbolic (see Diagram 0.10):

Diagram 0.10. World Literacy Dynamics, 1 – 1980 CE (%):
the fit between predictions of the hyperbolic model
and the observed data



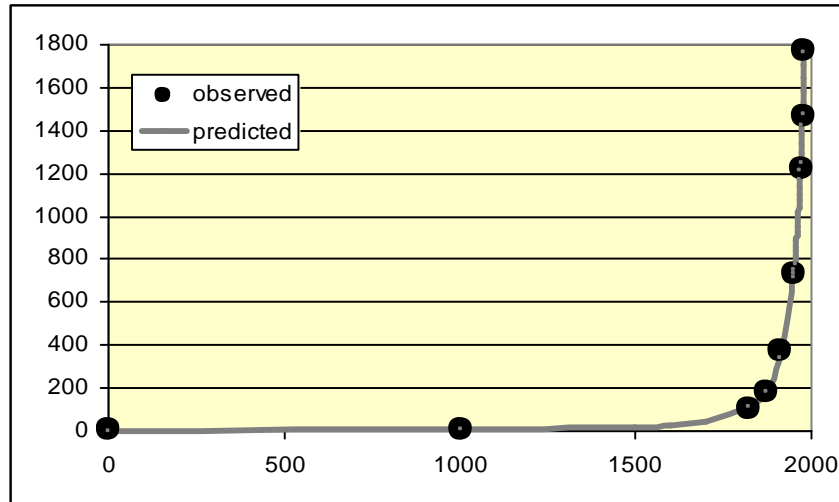
NOTE: $R = 0.997$, $R^2 = 0.994$, $p \ll 0.0001$. Black dots correspond to UNESCO/World Bank (2005) estimates for the period since 1970, and to Meliantsev's (1996, 2003, 2004a, 2004b) estimates for the earlier period. The grey solid line has been generated by the following equation:

$$l_t = \frac{3769.264}{(2040 - t)^2} .$$

The best-fit values of parameters (3769.264) and t_0 (2040) have been calculated with the least squares method.

The overall number of literate people is proportional both to the literacy level and to the overall population. As both of these variables experienced hyperbolic growth until the 1960s/1970s, one has sufficient grounds to expect that until recently the overall number of literate people in the world (L)²⁶ was growing not just hyperbolically, but rather in a quadratic-hyperbolic way (as was world GDP). Our empirical test has confirmed this – the quadratic-hyperbolic model describes the growth of the literate population of this planet with an extremely good fit indeed (see Diagram 0.11):

Diagram 0.11. World Literate Population Dynamics, 1 – 1980 CE (L , millions): the fit between predictions of the quadratic-hyperbolic model and the observed data



NOTE: $R = 0.9997$, $R^2 = 0.9994$, $p \ll 0.0001$. The black dots correspond to UNESCO/World Bank (2006) estimates for the period since 1970, and to Meliantsev's (1996, 2003, 2004a, 2004b) estimates for the earlier period; we have also taken into account the changes of age structure on the basis of UN Population Division (2006) data. The grey solid line has been generated by the following equation:

$$L_t = \frac{4958551}{(2033 - t)^2}.$$

The best-fit values of parameters (4958551) and t_0 (2033) have been calculated with the least squares method.

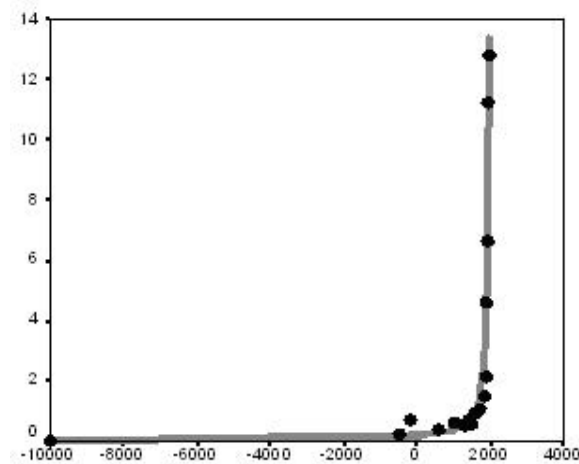
²⁶ Since literacy appeared, almost all of the Earth's literate population has lived within the World System; hence, the literate population of the Earth and the literate population of the World System have been almost perfectly synonymous.

Similar processes are observed with respect to world urbanization, the macro-dynamics of which appear to be described by the differential equation:

$$\frac{du}{dt} = bSu (u_{\text{lim}} - u), \quad (0.19)$$

where u is the proportion of the population that is urban, S is per capita surplus produced with the given level of the World System's technological development, b is a constant, and u_{lim} is the maximum possible proportion of the population that can be urban. Note that this model implies that during the "Malthusian-Kuznetsian" era of the blow-up regime, the hyperbolic growth of world urbanization must have been accompanied by a quadratic-hyperbolic growth of the urban population of the world, which is supported by our empirical tests (see Diagrams 0.12–13):

Diagram 0.12. World Megaurbanization Dynamics (% of the world population living in cities with > 250 thousand inhabitants), 10000 BCE – 1960 CE: the fit between predictions of the hyperbolic model and empirical estimates

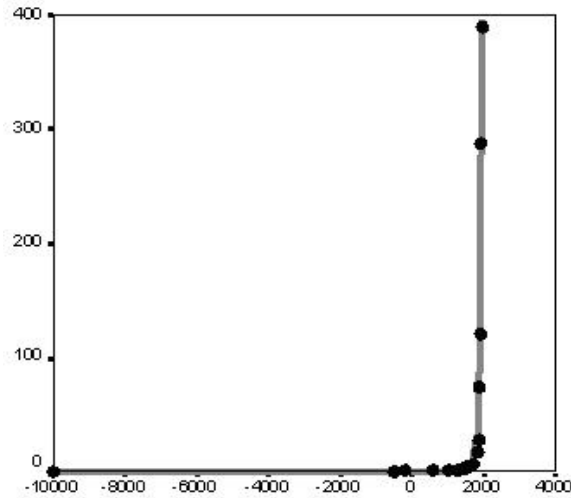


NOTE: $R = 0.987$, $R^2 = 0.974$, $p \ll 0.0001$. The black dots correspond to estimates of Chandler (1987), UN Population Division (2005), and White *et al.* (2006). The grey solid line has been generated by the following equation:

$$u_t = \frac{403.012}{(1990 - t)}$$

The best-fit values of parameters (403.012) and t_0 (1990) have been calculated with the least squares method. For a comparison, the best fit (R^2) obtained here for the exponential model is 0.492.

Diagram 0.13. Dynamics of World Urban Population Living in Cities with > 250000 Inhabitants (mlns.), 10000 BCE – 1960 CE: the fit between predictions of the quadratic-hyperbolic model and the observed data



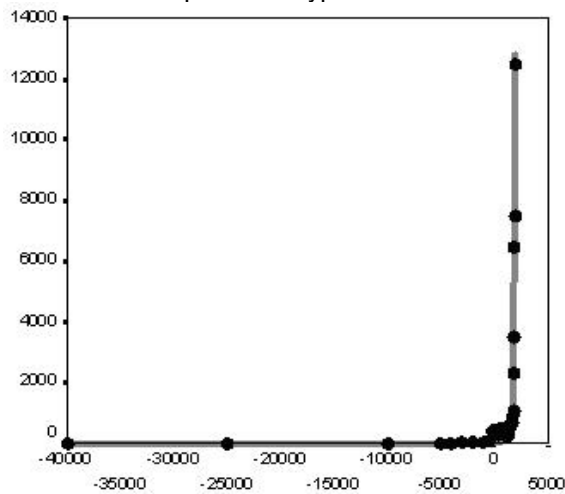
NOTE: $R = 0.998$, $R^2 = 0.996$, $p \ll 0.0001$. The black markers correspond to estimates of Chandler (1987), UN Population Division (2005), and White *et al.* (2006). The grey solid line has been generated by the following equation:

$$U_t = \frac{912057.9}{(2008 - t)^2}$$

The best-fit values of parameters (912057.9) and t_0 (2008) have been calculated with the least squares method. For a comparison, the best fit (R^2) obtained here for the exponential model is 0.637.

Within this context it is hardly surprising to find that the general macrodynamics of the size of the largest settlement within the World System are also quadratic-hyperbolic (see Diagram 0.14):

Diagram 0.14. Dynamics of Size of the Largest Settlement of the World (thousands of inhabitants), 10000 BCE – 1950 CE: the fit between predictions of the quadratic-hyperbolic model and the observed data



NOTE: $R = 0.992$, $R^2 = 0.984$, $p \ll 0.0001$. The black markers correspond to estimates of Modelski (2003) and Chandler (1987). The grey solid line has been generated by the following equation:

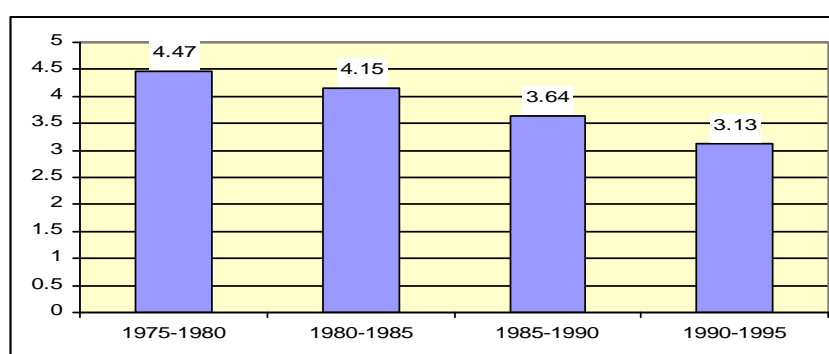
$$U_{\max t} = \frac{104020618,573}{(2040 - t)^2}$$

The best-fit values of parameters (104020618,5) and t_0 (2040) have been calculated with the least squares method. For a comparison, the best fit (R^2) obtained here for the exponential model is 0.747.

As has been demonstrated by cross-cultural anthropologists (see, *e.g.*, Naroll and Divale 1976; Levinson and Malone 1980: 34), for pre-agrarian, agrarian, and early industrial cultures the size of the largest settlement is a rather effective indicator of the general sociocultural complexity of a social system. This, of course, suggests that the World System's general sociocultural complexity also grew, in the "Malthusian-Kuznetsian" era, in a generally quadratic-hyperbolic way.

It is world literacy for which it is most evident that its hyperbolic growth could not continue, for any significant period, after the mid-1960s; after all, the literacy rate by definition cannot exceed 100 per cent just by definition. What is more, since the 1970s the saturation effect²⁷ described by our model started being felt more and more strongly and the rate of world literacy's growth began to slow (see Diagram 0.15):

Diagram 0.15. World Literacy Growth Dynamics, 1975 – 1995, the increase in percentage of adult literate population, by five-year periods



However, already before this, the hyperbolic growth of world literacy and of the other indicators of the human capital development had launched the process of diverging from the blow-up regime, signaling the end of the era of hyperbolic growth. As has been shown by us earlier (Korotayev, Malkov, and Khaltourina 2006a: 67–86), hyperbolic growth of population (as well as of cities, schools *etc.*) is only observed at relatively low (< 0.5 , *i.e.*, $< 50\%$) levels of world literacy. In order to describe the World System's demographic dynamics in the last decades (as well as in the near future), it has turned out to be necessary to extend the equation system (0.13)–(0.14) by adding to it equation (0.21), and by adding to equation (0.13) the multiplier $(1 - l)$, which results in equation (0.20), and produces a mathematical model that describes not only the hyperbolic de-

²⁷ On the ground, the saturation effect means, for example, that raising literacy from 98 to 100 per cent of the adult population would require much more time and effort than would raising it from 50 to 52 per cent.

velopment of the World System up to the 1960s/1970s, but also its withdrawal from the blow-up regime afterwards:

$$\frac{dN}{dt} = aSN(1-l), \quad (0.20)$$

$$\frac{dS}{dt} = bNS, \quad (0.14)$$

$$\frac{dl}{dt} = cSl(1-l). \quad (0.21)$$

We would like to stress that in no way are we claiming that the literacy growth is the only factor causing the demographic transition. Important roles were also played here by such factors as, for example, the development of medical care and social security subsystems. These variables, together with literacy, can be regarded as different parameters of one integrative variable, the human capital development index. These variables are connected with demographic dynamics in a way rather similar to the one described above for literacy. At the beginning of the demographic transition, the development of the social security subsystem correlates rather closely with the decline of mortality rates, as both are caused by essentially the same proximate factor – the GDP per capita growth. However, during the second phase, social security development produces quite a strong independent effect on fertility rates through the elimination of one of the main traditional incentives for the maximization of the number of children in the family.

The influence of the development of medical care on demographic dynamics shows even closer parallels with the effect produced by literacy growth. Note first of all that the development of modern medical care is connected in the most direct way with the development of the education subsystem. On the other hand, during the first phase of the demographic transition, the development of medical care acts as one of the most important factors in decreasing mortality. In the meantime, when the need to decrease fertility rates reaches critical levels, it is the medical care subsystem that develops more and more effective family planning technologies. It is remarkable that this need arises as a result of the decrease in mortality rates, which could not reach critically low levels without the medical care subsystem being sufficiently developed. Hence, when the need to decrease fertility rates reaches critical levels, those in need, almost by definition, find the medical care subsystem sufficiently developed to satisfy this need quite rapidly and effectively.

Let us recollect that the pattern of literacy's impact on demographic dynamics has an almost identical shape: the maximum values of population growth rates cannot be reached without a certain level of economic development, which

cannot be achieved without literacy rates reaching substantial levels. Hence, again almost by definition, the fact that the system reached the maximum level of population growth rates implies that literacy – especially of females – had attained such a level that its negative impact on fertility rates would cause population growth rates to start to decline. On the other hand, the level of development of both medical care and social security subsystems displays a very strong correlation with literacy (see Korotayev, Malkov, and Khaltourina 2006a: Chapter 7). Thus, literacy rate turns out to be a very strong predictor of the development of both medical care and social security subsystems.

Note that in reality, as well as in our model, both the decline of mortality at the beginning of the demographic transition (which caused a demographic explosion) and the decline of fertility during its second phase (causing a dramatic decrease of population growth rates) were ultimately produced by essentially the same factor (human capital growth); there is therefore no need for us to include mortality and fertility as separate variables in our model. On the other hand, literacy has turned out to be a rather sensitive indicator of the development level of human capital, which has made it possible to avoid including its other parameters as separate variables in extended macromodels (for more detail see Korotayev, Malkov, and Khaltourina 2006a: Chapter 7).

Model (0.20)-(0.14)-(0.21) describes mathematically the divergence from the blow-up regime not only for world population and literacy dynamics, but also for world economic dynamics. However, this model does not describe the slowdown of the World System's economic growth observed after 1973. According to the model, the relative rate of world GDP growth should have continued to increase even after the World System began to diverge from the blow-up regime, though more and more slowly. In reality, however, after 1973 we observe not just a decline in the speed with which the world GDP rate grows – we observe a decline in the world GDP growth rate itself (see, *e.g.*, Maddison 2001). It appears that model (0.20)-(0.14)-(0.21) would describe the recent world economic dynamics if the $(1 - l)$ multiplier were added not only to its first equation, but also to the second (0.14). This multiplier might have the following sense: the literate population is more inclined to direct a larger share of its GDP to resource restoration and to prefer resource economizing strategies than is the illiterate one, which, on the one hand, paves the way toward a sustainable-development trajectory, but, on the other hand, slows down the economic growth rate (*cp.*, *e.g.*, Liuri 2005).

Note that development, according to this scenario, does not invalidate Kremer's technological growth equation (0.12). Thus, the modified model does imply that the World System's divergence from the blow-up regime would stabilize the world population, the world GDP, and some other World-System development indicators (*e.g.*, urbanization and literacy as a result of saturation, *i.e.*, the achievement of the ultimate possible level); technological growth, however, will continue, though in exponential rather than hyperbolic form.

Due to the continuation of technological growth, the ending of growth in the world's GDP will not entail a cessation of growth in the standard of living of the world's population. A continuing rise in the world's standard of living is most likely to be achieved due to the so-called "Nordhaus effect" (Nordhaus 1997). The essence of this effect can be spelled out as follows: imagine that you are going to buy a new computer and plan to spend \$1000 on this. Now imagine what computer you would have been able to buy with the same \$1000 five years ago. Of course, the computer that you will be able to buy with \$1000 now will be much better, much more effective, much more productive *etc.* than the computer that you could have bought with the same \$1000 five years ago. However, open a current World Bank handbook and you will see that the present-day \$1000, in terms of purchasing power parity (PPP), constitutes a significantly smaller sum than did the \$1000 of five years ago. The point is that traditional measures of economic growth (above all, the GDP as measured in international PPP dollars) reflect less and less the actual growth of the standard of living (especially in more developed countries). Imagine a firm that in 2001 produced 1 million computers and sold them at \$1000 a piece, in 2006 the same firm produced 1 million 100 thousand new, much more effective computers, but still sells them (due to increasing competition) at \$1000 a piece (let us also imagine that the firm has managed to reduce production costs and thus increased both its profits and employees' salaries). How will this affect GDP, both in the country in which the firm operates, and in the world as a whole? In fact, the effect is most likely to be exactly zero. In 2006 the firm produces computers for a total price of 1100 million 2006 international PPP dollars. However, the World Bank will recalculate this sum into 2001 international PPP dollars and will find out that 1100 million 2006 international PPP dollars equal just 1000 million 2001 international PPP dollars. Thus, technological progress sufficient to raise the level of life of a significant number of people will in no way affect the World Bank GDP statistics, according to which it will appear that the above-mentioned technological advance has led to GDP increase at neither the country nor the world level.

The point is that the traditional GDP measures of production growth work really well when they are connected with the growth of consumption of scarce resources (including labor resources); however, if the production growth takes place without an increase in the consumption of scarce resources, it may well go undetected. The modified macromodel predicts such a situation when the World System's divergence from the blow-up regime will have resulted in the cessation of the resource-consuming World GDP production in its traditional measures, accompanied by the transition to exponential (in place of hyperbolic) growth of technology through which an increasing standard of living will be achieved without the growth of scarce-resource consumption.

Because the macrodynamics of the World System's development obey a set of rather simple laws having extremely simple mathematical descriptions, the

macroproportions between the main indicators of that development can be described rather accurately with the following series of approximations:

$$N \sim S \sim l \sim u, \\ G \sim L \sim U \sim N^2 \sim S^2 \sim l^2 \sim u^2 \sim SN \sim etc.,$$

where (let us recollect) N is the world population; S is per capita surplus produced, at the given level of the World System's technological development, over the "hungry survival" level m that is necessary for simple (with zero growth) demographic reproduction; l is world literacy, the proportion of literate people among the adult (> 14 year old) population of the world; u is world urbanization, the proportion of the world population living in cities; G is the world GDP; L is the literate population of the world; and U is the urban population of the world. Yes, for the era of hyperbolic growth the absolute rate of growth of N (but, incidentally, also of S , l and u) in the long-run is described rather accurately²⁸ as kN^2 (Kapitza 1992, 1999); yet, with a comparable degree of accuracy it can be described as k_2SN , k_3S^2 or (apparently with a somehow smaller precision) as k_4G , k_5L , k_6U , k_7l^2 , k_8u^2 , etc.

It appears important to stress that the present-day decrease of the World System's growth rates differs radically from the decreases that inhered in oscillations of the past. This is not merely part of a new oscillation; rather, it is a phase transition to a new development regime that differs radically from the one typical of all previous history. Note, first of all, that all previous cases of reduction of world population growth took place against the background of catastrophic declines in the standard of living, and were caused mainly by increases in mortality as a result of various cataclysms – wars, famines, epidemics; and that after the end of such calamities the population, having restored its numbers in a relatively rapid way, returned to the earlier hyperbolic trajectory. In sharp contrast, the present day decline of the world population's growth rate takes place against the background of rapid economic growth and is produced by a radically different cause – the decline of fertility rates that is occurring precisely because rising standards of living for the majority of the World System's population have meant the growth of education, health care (including various methods and means of family planning), social security, etc. Decrease in the rate of growth of literacy and urbanization was not infrequent in the earlier epochs either; but in those epochs it was connected with economic decline, whereas now it takes place against the contrary background of rapid economic growth, and is connected to the closeness of the saturation level. Earlier declines, we might say, reflected a deficit of economic resources, whereas the present one reflects their abundance.

It appears necessary to stress that the models discussed above have been designed to describe long-term ("millennial") trends, whereas when we analyze social macrodynamics at shorter ("secular") time scales we also have to take in-

²⁸ However, for u the fit of this description appears to be smaller than for the rest of variables.

to account its cyclical (as well as stochastic) components; it is these components that will be the main task of the present part of our *Introduction to Social Macrodynamics*. To begin with, the actual dynamics typical for agrarian political-demographic cycles are usually the opposite of those that are theoretically described by "millennial" models and actually observed at the millennial scale. For example, as we shall see below, during agrarian political-demographic cycles the population normally grew much faster than technology, which naturally resulted in Malthusian dynamics: population growth was accompanied not by increase, but by decrease of per capita production, usually leading to political-demographic collapse and the start of a new cycle.

In Chapter 1, we shall review available mathematical models of political-demographic cycles. In Chapter 2, we shall consider in more detail political-demographic cycles in China, where long-term population dynamics have been recorded more thoroughly than elsewhere. In Chapter 3 we shall present our own model of pre-Industrial political-demographic cycles. Finally, in Chapter 4 we shall consider the interaction between long-term trends and cyclical dynamics.