From *Introduction to Social Macrodynamics. Secular Cycles and Millennial Trends in Africa* by Andrey Korotayev and Daria Khaltourina. Moscow: KomKniga/URSS, 2006. Pp. 116–143.

Appendix

Cyclical Dynamics and Mechanisms of Hyperbolic Growth

The compact macromodels, which we discussed in the Introduction to this volume, specify the most general mechanisms of the world population growth. However, we also need to know how these macrotrends are produced on a more specific level.

In the first part of our *Introduction to Social Macrodynamics* (Korotayev, Malkov, and Khaltourina 2006a: 92–104) we have discussed the most evident specific mechanism accounting for both the hyperbolic population growth[1] in 1850–1962/3 and inverse-hyperbolic (logistic) trend afterwards – the one associated with the demographic transition (*e.g.*, Chesnais 1992; Kapitza 1999). As is well known, during the first phase of demographic transition a rather sharp decline in mortality rates is observed. This is followed by decline in fertility rates (through the introduction of family planning practices and technologies as a proximate cause), but with a substantial time lag. As a result for considerable periods of time we observe pronounced trends towards the rise of the population growth rates against the background of growing population. This, of course, exactly produces a hyperbolic effect – the higher is the population (N), the higher is the population growth rate (r). Since the 19th century more and more populations of the world entered the demographic transition. Up until the 1960s the number of populations which entered the 2nd phase of demographic

[1] Let us recollect that hyperbolic population growth implies that the absolute population growth rate is proportional to the square of population (unlike exponential growth, for which the absolute growth rate is lineally proportional to population). Thus, with exponential growth if, at a world population level of 100 million, the absolute annual growth rate was 100 thousand people a year, at a level of 1 billion level it will be 1 million people a year (a 10-fold growth of population leads to an equivalent 10-fold increase in the absolute population growth rate). For hyperbolic growth, if, at the world population level of 100 million, the absolute annual growth rate was 100 thousand people a year, at a level of 1 billion it will be 10 million people a year (the 10-fold growth of population leads to a 100-fold increase in the absolute population growth rate). Note that the *relative* population growth rate will remain constant with exponential growth (0.1% in our example), whereas it will be lineally proportional to the absolute population level with hyperbolic growth (in our example, population growth by a factor of 10 leads to an increase in the relative annual growth rate also by a factor of 10, from 0.1% to 1%). Such a growth is hyperbolic, just because it implies that the relative population growth rate is proportional to population size: $dN/dt \div N = kN$. If we multiply both sides of this equation by N, we will get $dN/dt = kN^2$, whereas the solution of this differential equation is just the hyperbolic formula $N_t = C/(t_0 - t)$, where $C = 1/k$ (see the Introduction).

transition did not compensate for the hyperbolic growth of the 1st phase populations, hence, the hyperbolic growth trend was characteristic not only for individual populations, but also for the world population as a whole.

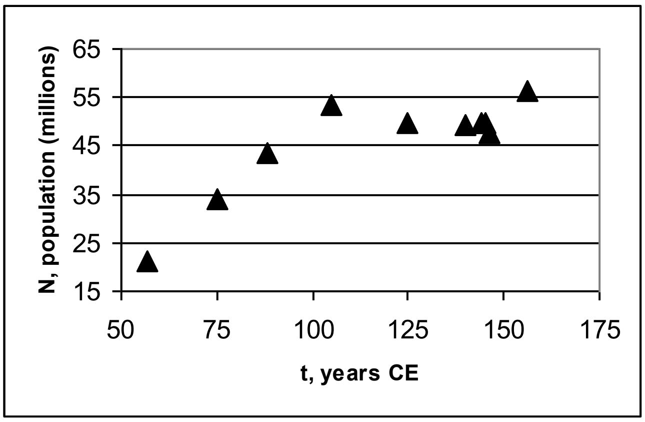
The only problem with the mechanism of demographic transition is that it is impossible to use it to account for the hyperbolic growth trend in the pre-19th century history of the humankind.

In fact, against the background of our earlier discussion of the pre-industrial cyclical dynamics it should not appear strange that the presence of hyperbolic population growth trend in the pre-industrial period of human history looks quite counterintuitive for those specialists who deal with particular historical demographies. Indeed, whenever we manage to acquire any quantitative data on pre-Modern population dynamics for any particular countries on a century time scale (and this happens infrequently), we tend to observe just the contrary trend – the higher the population, the lower its relative growth rate. Let us return[2] to the historical population dynamics in China.

World population growth and historical demographic dynamics of China

For example, the data of the Chinese census of the Late (Eastern) Han period (in fact, the first period and place in human history, for which we have any direct and systematic population data) looks as follows (see Diagrams A1–2, and Table A1):

Diagram A1. Population Dynamics of China during the Eastern Han Period according to Contemporary Census Data (millions)



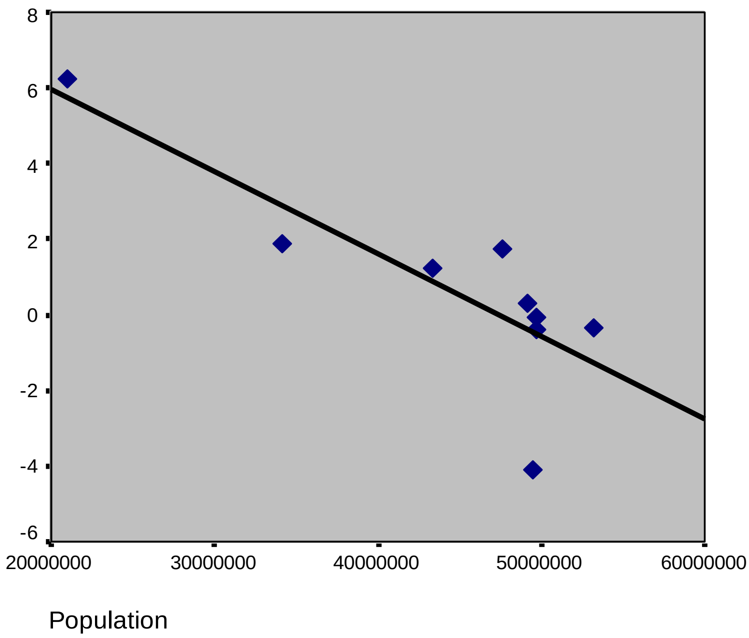
SOURCES: Bielenstein 1947: 126; 1986: 240–2; Durand 1960: 216; Loewe 1986c: 485; Zhao and Xie 1988: 536.

[2] For our earlier detailed treatment of the historical population dynamics of China see the previous volume of our *Introduction to Social Macrodynamics* (Korotayev, Malkov, and Khaltourina 2006b: Chapter 2).

Table A1. Population Numbers and Growth Rates in China during the Eastern Han Period according to Contemporary Census Data

Census year	*Population registered by census*	*Population growth rate for subsequent intercensus period (%%)*
57	21008000	6.25
75	34125000	1.86
88	43356000	1.22
105	53256000	–0.34
125	49691000	–0.07
140	49150000	0.29
144	49731000	–0.41
145	49524000	–4.1
146	47567000	1.73
156	56487000	

Diagram A2. Correlation between Population Numbers and Population Growth Rates for Eastern Han China



NOTE: $r = -0.82$, $p = 0.007$.

The extremely high population growth rate for 57–75 CE is, no doubt, a result of underregistration in 57 CE. Otherwise, the overall picture is quite clear, logical, and precisely contrary to the one implied by the hyperbolic growth models – the higher the population, the lower the population growth rates.

As we could see above, a rather convincing explanation for this pattern recurrently found in pre-industrial populations is provided by demographic cycle models – at the initial phases of such cycles resources are abundant, consumption levels are high, and thus, the population growth rates are also high; with population growth per capita acreage decreases, which against a context (typically observed within pre-industrial agrarian systems) of stable, or very slowly growing subsistence technology levels would normally lead to decreasing per capita food production and food consumption, and to decreasing population growth rates, which after reaching the ceiling of the carrying capacity could eventually drop to zero, or even negative values (see, *e.g.*, Abel 1974, 1980; Postan 1973; Kuhn 1978; Mugruzin 1986, 1994; Usher 1989; Kul'pin 1990; Chu and Lee 1994; Huang 2002; Nefedov 2003, 2004, 2005; Turchin 2003b, 2005a, 2005b; Nepomnin 2005; Turchin and Korotayev 2006; Korotayev, Malkov, and Khaltourina 2006b).

Within the East Han cycle the population growth slowed down dramatically after the population approached 60 million. Note that after the population had approached this level very closely, the system experienced demographic collapse (starting in 186 CE). It is remarkable that demographic collapses also occurred at the same level during the West Han (in the early 1st century CE), Sui (in the early 7th century) and Early Tang (mid 8th century) cycles (see, *e.g.*, Bielenstein 1947: 126, 1986: 240; Durand 1960: 216, 223; Loewe 1986b: 206; Nefedov 1999e: 5; 2003: Fig. 10; Lee Mabel Ping-hua 1921: 436; Wechsler 1979a; Wright 1979: 128–49; Zhao and Xie 1988: 536–7).

Note also that Sung China experienced all the pre-collapse symptoms after its population approached the same level in the early 11th century – famines, rising rebellions *etc.* (*e.g.*, Lee Mabel Ping-hua 1921: 281–2; Smolin 1974: 311–57; Nefedov 1999e: 9, *etc.*).[3] All this, of course, suggests 60 million as an effective ceiling of the carrying capacity of land for 1st millennium CE China. This ceiling was radically raised only in the 11th century through the Sung "green revolution" (*e.g.*, Ho 1956, 1959: 169–70, 177–8; Shiba 1970: 50; Bray 1984: 79, 113–4, 294–5, 491–4, 597–600; Mote 1999: 165).

Another interesting observation on population dynamics during the East Han cycle is that, according to Chinese census in 57–105 CE the average annual population growth rate was *c.*2%. Against the background of data on fairly high life expectancies in China during the phases of high population growth (*e.g.*, Harrell and Pullum 1995: 148; Liu 1995: 118–9; Heijdra 1994, 1998: 437) we

[3] However, the Sung mid-phase demographic crisis resulted not in a demographic collapse, but in the non-catastrophic solution of the crisis through the radical raising of the carrying capacity of land ceiling (see below).

do not see why the possibility of 2% annual population growth rates during initial phases of Chinese demographic cycles could be completely excluded.

However, even if we take a much more conservative estimate of the Chinese population growth rates in the second half of the 1st century CE as being *c.*1.5% (*e.g.,* Durand 1960: 216–21), we will still get a value of the world population growth rate far exceeding the one attested for the last 50 years (1750–1800) of the pre-industrial period, < 0.45% (Kremer 1993: 683). This is accounted for by the fact that at the end of the 1st century the population of China constituted around one third of the world population and the point that the Roman Empire (encompassing by that moment almost another third of the world population) also experienced a significant demographic growth during the 1st century (see, *e.g.*, Turchin 2003: 162).

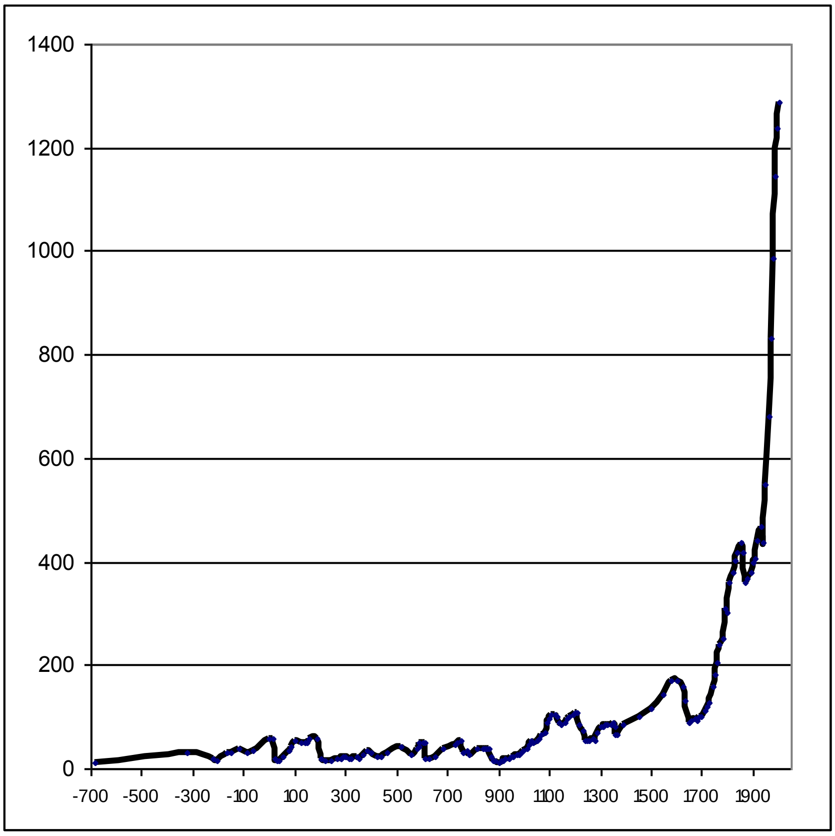
This, of course, suggests that 0.45% attested as the average annual world population growth rate for the second half of the 18th century (see, *e.g.*, Kremer 1993) was not only significantly lower than the one achieved by particular (and rather significant) regional populations long before that time, but also that long before the 18th century the world population growth rates could be equal or higher than the ones attested at the end of the pre-industrial epoch. This also suggests that the hyperbolic effect might have been created not by the absolute increase in the population growth rates during the pre-industrial demographic expansion periods, but rather by the changes in lengths and spacing of those periods.

Indeed, the actual increase in annual population growth rates in pre-industrial era would imply the growth of life expectancies at birth, whereas the evidence does not indicate any significant growth of life expectancies between the Neolithic and Industrial revolutions, suggesting rather a general trend towards its decline in the Neolithic and Post-Neolithic epoch up to the Modern Age when life expectancies started to grow significantly marking the beginning of demographic transition (*e.g.*, Lee and de Vore 1968; Mel'jantsev 1996; Kozintsev 1980; Storey 1985; Fedosova 1994; Cohen 1977, 1987, 1989, 1995, 1998; Cohen and Armelagos 1984; Ember and Ember 1999: 152–3, *etc.*) . In the rest of this appendix we shall demonstrate how the hyperbolic trend of world population growth in the pre-Modern Age could co-exist with the absence of an increased annual growth rate during the pre-industrial demographic expansion (as well as, by definition, stagnation) phases.

Let us consider now the overall of demographic dynamics in China. Against the background of what has been mentioned above it might be somehow counterintuitive to find that we do observe a hyperbolic growth trend for this population. Naturally this trend turns out to be more pronounced if we take into account the last 150 years of Chinese demographic history (see Diagrams A3–5):[4]

[4] We use the estimates of historical Chinese population surveyed in the previous issue of our *Introduction to Social Macrodynamics* (Korotayev, Malkov, and Khaltourina 2006b: Chapter 2).

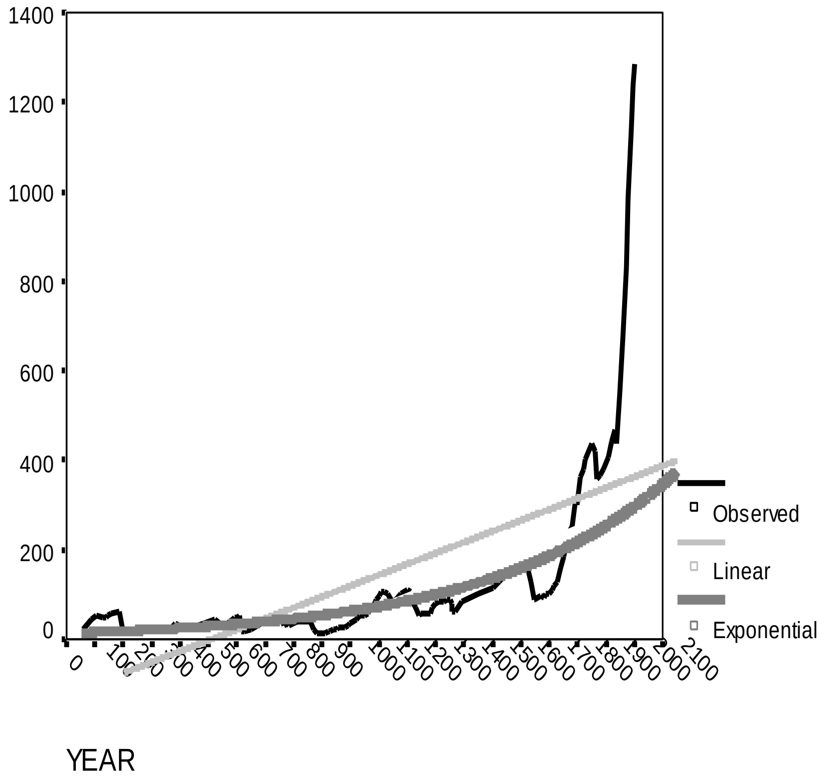
Diagram A3. Population Dynamics of China, millions (700 BCE – 2003 CE)



It is easy to see here a pattern of interplay of cyclical and trend dynamics. However, what kind of trend do we observe here? Linear regression suggests a statistically significant ($p < 0.001$) relationship with $R^2 = 0.398$.[5] Exponential regression produces an even stronger result with $R^2 = 0.685$ ($p < 0.001$), see Diagram A4:

[5] All regressions for pre-industrial and industrial periods combined were calculated for years 57–2003.

Diagram A4. Curve Estimations for Chinese Population Dynamics, millions, 57 – 2003 CE (linear and exponential models)

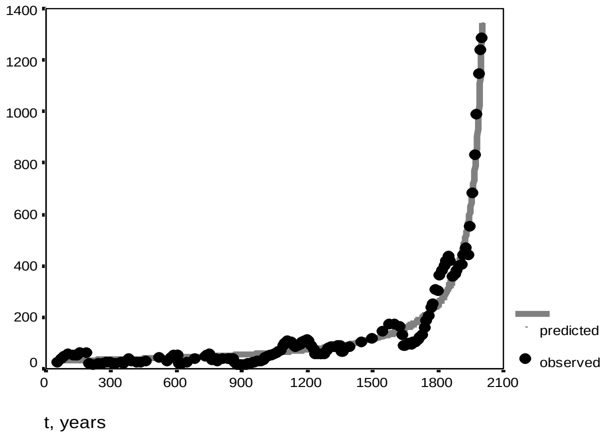


NOTES: the thin black line corresponds to the observed population dynamics surveyed in Chapter 2 of the previous issue of our *Introduction to Social Macrodynamics* (Korotayev, Malkov, and Khaltourina 2006b). *Lineal regression*: R = 0.631, R^2 = 0.398, p < 0.001. The respective best-fit thin light grey line has been generated by the following equation: $N_t = 0.2436t - 124.25$. *Exponential regression*: R = 0.828, R^2 = 0.685, p < 0.001. The respective best-fit thick dark grey line has been generated by the following equation: $13.3575 \times e^{0,0015t}$. The best-fit values of parameters have been calculated with the least squares method.

However, a simple hyperbolic growth model produces a much better fit with the observed data (R^2 = 0.968, p << 0.001[6]), see Diagram A5:

[6] In fact, to be exact, statistical significance of the fit in this case reaches an astronomical level of 1.67×10^{-19}.

Diagram A5. Population Dynamics of China (57 – 2003 CE), millions, correlation between the observed values and the ones predicted by a hyperbolic growth model



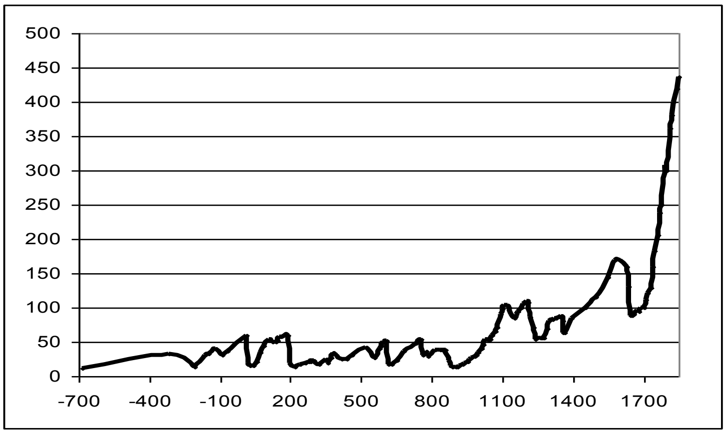
NOTE: R = 0.984, R^2 = 0.968, p = 1.7×10^{-19}. The black markers correspond to empirical estimates surveyed in Chapter 2. The grey solid line has been generated by the following equation:

$$N_t = \frac{63150.376}{2050 - t}.$$

The best-fit values of parameters C (63150.376) and t_0 (2050) have been calculated with the least squares method.

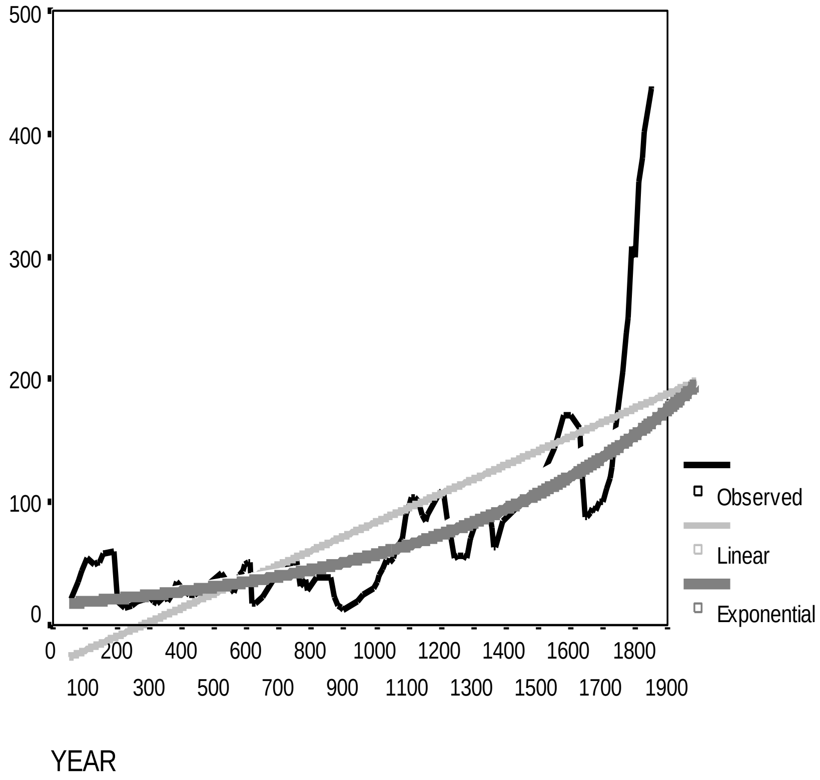
Yet, even if we consider only the pre-Modern history of China (up to 1850), we will still find the hyperbolic growth trend for this part of Chinese history too (see Diagrams A6–8):

Diagram A6. Population Dynamics of Pre-Modern China (700 BCE – 1850 CE)



What kind of trend do we observe here? Linear regression again suggests a statistically significant (p < 0.001) relationship with R^2 = 0.469. Exponential regression again produces an even stronger result with R^2 = 0.593 (p < 0.001), see Diagram A7:

Diagram A7. Curve Estimations for Pre-Modern Chinese Population Dynamics, millions, 57 – 1850 CE (linear and exponential models)



NOTES: the thin black line corresponds to the observed population dynamics surveyed in Chapter 2 of the previous issue of our *Introduction to Social Macrodynamics* (Korotayev, Malkov, and Khaltourina 2006b). *Lineal regression*: $R = 0.689$, $R^2 = 0.469$, $p < 0.001$. The respective best-fit thin light grey line has been generated by the following equation: $N_t = 0.1098t - 27.97$. *Exponential regression*: $R = 0.770$, $R^2 = 0.593$, $p < 0.001$. The respective best-fit thick dark grey line has been generated by the following equation: $16.9785 \times e^{0{,}0012t}$. The best-fit values of parameters have been calculated with the least squares method.

However, a simple hyperbolic growth model once more produces a much better fit with the observed data ($R^2 = 0.884$, $p << 0.001$[7]), see Diagram A8:

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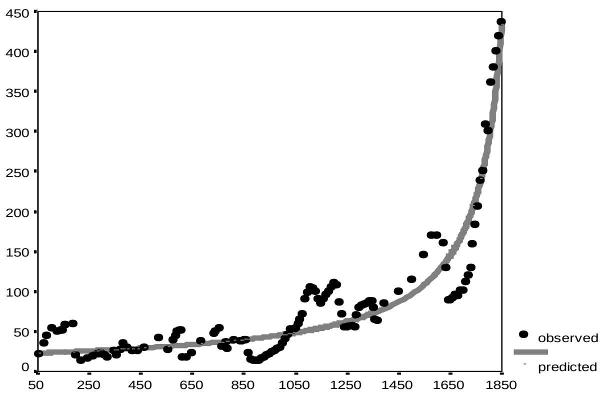
[7] To be exact, statistical significance of the fit in this case again reaches an astronomical level (2.8×10^{-19}).

Diagram A8. Population Dynamics of Pre-Modern China (57 – 1850 CE), millions, correlation between the observed values and the ones predicted by a hyperbolic growth model



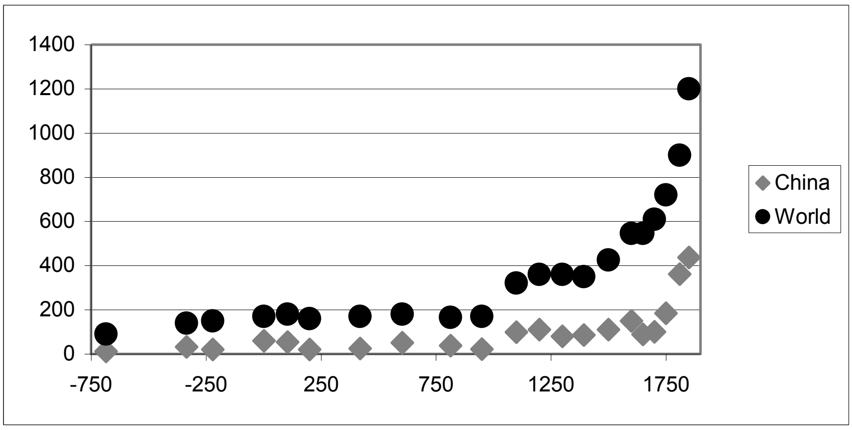
NOTE: $R = 0.94$, $R^2 = 0.884$, $p = 2.8 \times 10^{-19}$. The black markers correspond to empirical estimates surveyed in Chapter 2. The grey solid line has been generated by the following equation:

$$N_t = \frac{33430.518}{1915 - t}.$$

The best-fit values of parameters C (33430) and t_0 (1915) have been calculated with the least squares method.

Note that the population dynamics of China for the pre-Modern period correlate rather well with the population dynamics of the world (see Diagram A9):

Diagram A9. Population Dynamics of the World and China, millions, 700 BCE – 1850 CE

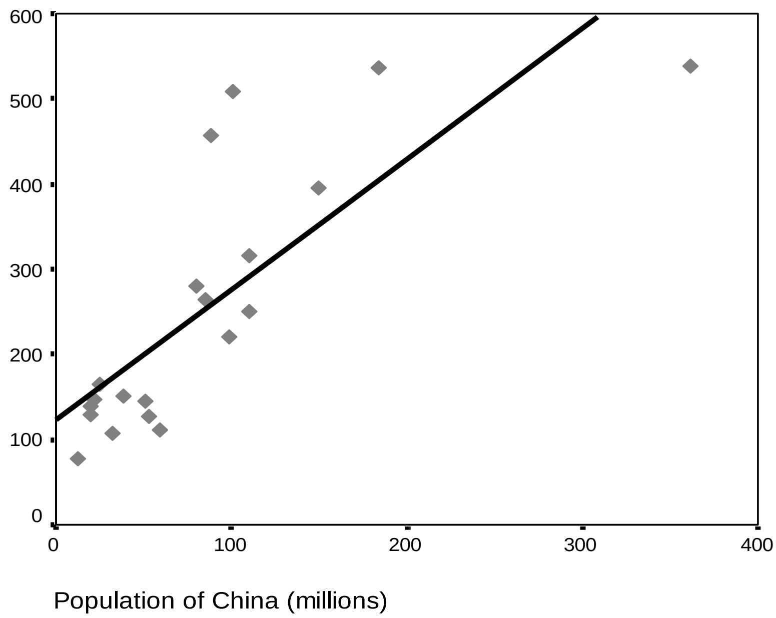


NOTE: The world population data here and elsewhere are from Kremer 1993: 683 (the other sources consulted are: Thomlinson 1975; Durand 1977; McEvedy and Jones 1978: 342–51; Haub 1995: 5; Biraben 1980; U.S. Bureau of the Census 2004; UN Population Division 2004). The China population data are from Bielenstein 1947, 1986; Durand 1960; Ho 1959; Lee 1921;

Mel'jantsev 1996; Nefedov 2003, 2004; Zhao and Xie 1988 (surveyed in the previous issue of our *Introduction to Social Macrodynamics* [Korotayev, Malkov, and Khaltourina 2006b]).

As could be expected from the graph above, the population of China for this period correlates very well (R = 0.907, R^2 = 0.822, p < 0.001) with the world population. There is, of course, a very significant autocorrelation component here, which can be easily controlled, if we compare the dynamics of Chinese population with the one of the rest of the world. It is highly remarkable that in this case the correlation remains very strong and significant (see Diagram A10):

Diagram A10. Population of China *vs.* Population of the Rest of the World (700 BCE – 1850 CE)



NOTE: R = 0.793, R^2 = 0.628, p < 0.001.

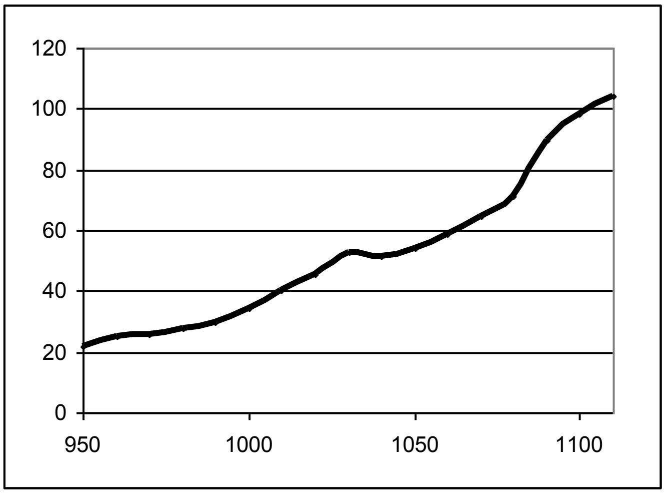
This, of course, suggests that the structure of Chinese pre-industrial population dynamics (known in more detail than for any other part of the world) could reveal a lot with respect to the world population dynamics in the pre-Modern period.

As can be easily seen in Diagram A6, the upward hyperbolic trend is created by just 4 relatively long periods of population growth accompanied by series of carrying capacity increasing innovations – during West Han, Sung, Ming and Qing dynasties. As the census data for China are only available since 2 CE, more or lest reliable dynamics of Chinese population during West Han (206

BCE – 9 CE) remains unknown, the shape of the West Han population curve in diagrams above is based on estimates by Zhao and Xie (1988: 536), which could hardly be used for any exact analysis. The only thing, which seems to be clear is that during this period the Chinese population did not only manage to restore its numbers to the level preceding the demographic collapse of the Qin – Han transition, but grew substantially (due to a series of the carrying capacity enhancing innovations [see, *e.g.*, Bray 1984]) over the pre-Han level (which is not known exactly either, *e.g.*, according to Zhao and Xie's [1988: 536] estimates, it was around 20–32 million) up to *c.* 60 million, whereas afterwards (up to the 11th century) the Chinese population oscillated below this level. There do not seem to be any factual grounds to estimate the population growth rates during the restoration and "pure growth" phases[8] of the West Han cycle.

During the Sung cycle a relatively high population growth was observed for about a century, which could be roughly split into two parts, or phases – a restoration phase, and a pure growth phase (see Diagram A11):

Diagram A11. Sung China Population Dynamics (millions)



[8] The following terminology is used throughout this appendix. The demographic cycle phase when the population is restored to the pre-collapse (usually close to the carrying capacity of land ceiling) level is denoted as a "restoration phase", the phase when population grows over this level (through the introduction of the carrying capacity enhancing innovations) is denoted as a "pure growth phase". Though restoration phases are observed in all Chinese demographic cycles, the pure growth phases are only found within the West Han, Sung, Ming, and Qing cycles, and not observed in the East Han, Sui, Early and Late Tang, as well as Yüan cycles. During the Late Tang and Yüan cycles demographic collapses took place before the population reached the carrying capacity of land ceiling.

The Sung restoration phase occupied the late 10th – early 11th centuries and lasted until the 1030s, when the population grew over 50 million and approached the carrying capacity limit, which resulted in a political-demographic crisis with all the pre-collapse symptoms. However, Sung China reacted to this crisis in a rather adequate way, through the well-coordinated introduction of a series of the carrying capacity increasing innovations (*e.g.*, Ho 1956, 1959: 169–70, 177–8; Shiba 1970: 50; Bray 1984: 79, 113–4, 294–5, 491–4, 597–600; Mote 1999: 165). As a result, the carrying capacity in China was raised about twice, and afterwards the Chinese population oscillated well above the 1st millennium CE level.

During the pure growth phase (*c.*1050–1110) the average annual growth rate was *c.*1.1%, which was somehow lower than the growth rate achieved during the East Han restoration phase (57–105 CE), but was still much higher than the highest pre-industrial world population growth rate evidenced for 1750–1800 CE.

The official Ming census records give rather lower figures, indicating that the population grew up to 60.5 million by 1393 and then fluctuated between slightly more than 50 million (1431–1435, 1487–1504) up to 63–65 million (1486, 1513, 1542–1562); in 1602 it was 56.3 million, in 1620–1626 it was 51.7 million (*e.g.*, Durand 1960: 231–2). There is a consensus that the actual population of Ming China was much higher.

What is more, this appears to have been clear for the Ming Chinese themselves:

"The official census records were hopelessly out of touch with demographic reality. The compiler of a Zhejiang gazetteer of 1575 insisted that the number of people off the official census registers in his county was three times the number on. A Fujian gazetteer of 1613 similarly dismissed the impression of demographic stagnation conveyed by the official statistics: 'The realm has enjoyed, for some two hundred years, an unbroken peace which is unparalleled in history,' the editor pointed out. 'During this period of recuperation and economic development the population should have multiplied several times since the beginning of the dynasty. It is impossible that the population should have remained stationary.' A Fujian contemporary agreed: 'During a period of 240 years when peace and plenty in general have reigned [and] people no longer know what war is like, population has grown so much that it is entirely without parallel in history.' Another official in 1614 guessed that the increase since 1368 had been fivefold. China's population did not grow between 1368 and 1614 by a factor of five, but it certainly more than doubled" (Brook 1998: 162).

Thus, nobody appears to doubt that the actual population of Ming China was much higher than is indicated by the Ming census (what is more, many Ming Chinese do not seem to have had doubts about this either); however, there is no consensus at all as regards how much higher it was.

The lowest estimate is 100 million (Zhao and Xie 1988: 540). Most experts suggest for the end of the Ming much higher figures: 150 million (Ho 1959:

264), 120–200 million (Perkins 1969: 16), 175 million (Brook 1998: 162), 200 million (Chao 1986: 89), or even 230–290 million (Heijdra 1998: 438–40; Mote 1999: 745), though the last figures appear to be overestimations (see, *e.g.*, Marks 2002). In any case, as we have argued in the previous volume of our *Introduction to Social Macrodynamics* (Korotayev, Malkov, and Khaltourina 2006b), there does not appear to be much doubt that the Ming cycle included both restoration and pure growth phases, but there does not seem to be sufficient grounds to estimate population growth rates during the Ming pure growth phase. As the highest population level achieved during the Ming period remains unknown, it is difficult to demarcate the boundary between the restoration and pure growth phases within the Qing cycle.

If we accept as such the figure of 150 million, then the Qing pure growth phase would start in *c.* 1740. The pure growth phase would then last for 110 years, during which the average growth would be *c.* 1%.[9]

Thus, there are no grounds to believe that the population growth rates during later phases of pure growth were higher than during pure grow phases of earlier cycles. The available evidence rather suggests that as soon as free resources became available (due either to previous depopulation or to series of carrying capacity increasing innovations); the Chinese population grew with fairly similar rates up to the point when the available resources were exhausted.

How could this condition coexist with the hyperbolic growth trend? We shall try to start answering this question below.

Pure growth phase lengthening mechanism

Let us now try to apply the compact macromodels of hyperbolic growth (see the Introduction, and Korotayev, Malkov, and Khaltourina 2006a) to Chinese data. Of course, the main problem with the direct application of these models to the Chinese data (and, as we shall see soon, to the world data, in general, as well) is that we do not have evidence for the actual systematic rise of population growth rate during later stagnation and expansion periods of Chinese demographic history in comparison with earlier ones. Thus, this model has to be translated into more specific mechanisms explaining how hyperbolic growth could appear against a background of absence of increase in population growth rates during pure growth phases.

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[9] Note that this rate was significantly higher than the one attested for the world population growth in the contemporary period. Naturally, as a result, the proportion of Chinese population to the whole population of the world grew very significantly from 16.5% in 1700 to > 36% in 1850. This proportion experienced a sharp decline in 1850–1870, as a result of the "Taiping" demographic collapse with the total human life losses as high, as 118 million (see, *e.g.*, Huang 2002: 528) against the background of a rather high population growth rates attested in most other parts of contemporary world.

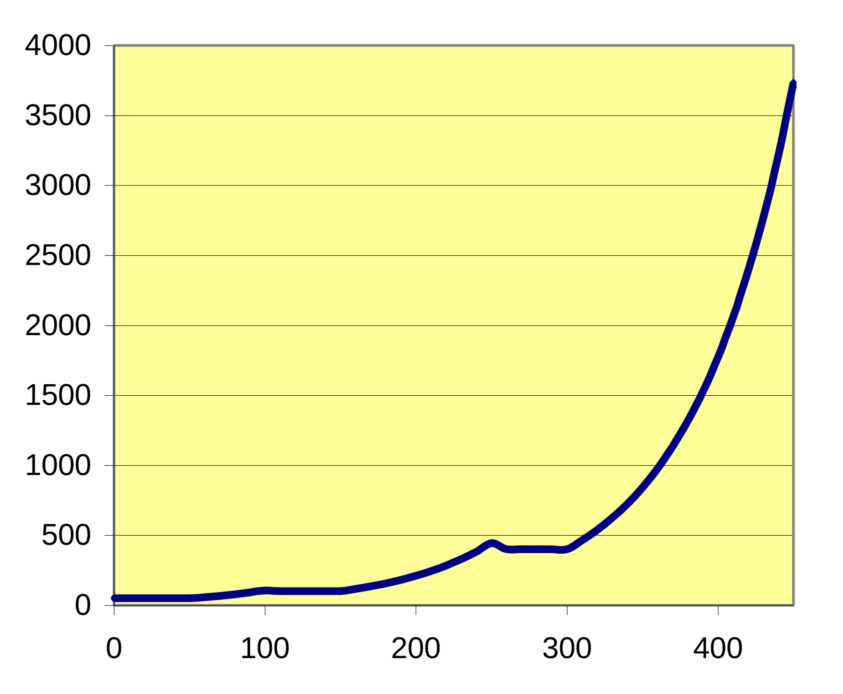
One possible prediction which could be made on the basis of the compact macromodels is as follows: the higher the population at the beginning of the given pure growth phase, the greater the number of innovations that will be made during this phase, the higher the level, to which the carrying capacity (and, thus, the population) will grow. Note that even if we assume that the population growth rate during the pure growth phase is constant, this will still produce a certain hyperbolic effect.

The immediate logic of the compact macromodels suggests that the rate of carrying capacity increase during the pure growth phase would be proportional to the square of population at the beginning of the phase. This will result in the following dynamics.

In our first auxiliary model we assume that the pure growth phases are separated by equal 50-year interphases.[10] During each pure growth phase the population growth is assumed to be 1.5 %. After the first 50-year interphase the first pure growth phase starts (at 100 million level), during which the carrying capacity is raised twice, whereas the population will grow with 1.5 % rate for about 50 years, after which the new limit of the carrying capacity is reached, and the population stabilizes at the 200 million level. A new pure growth phase starts after a 50-year interphase. This time the population growth starts from 200 million, so the logic of compact macromodels would suggest that during this phase the carrying capacity would be raised 4 times, which would make it possible for the population to grow up to 800 million level, thus securing *c.* 100 years of population growth with 1.5% annual rate. During the next growth phase we would expect the carrying capacity to be raised 64 times, which would secure 1.5% a year growth for 300 hundred years. The population dynamics for the 450 years just described starting with zero interphase would look as follows (see Diagrams A12–13 and Table A2):

[10] We denote periods of innovations leading to the absolute increases in carrying capacity and pure growth of population followed by stagnation periods as "developmental cycles"; we subdivide developmental cycle into "growth phase" and "stagnation phase" (= "interphase").

Diagram A12. Dynamics Produced by the Pure Growth Phase Lengthening Mechanism

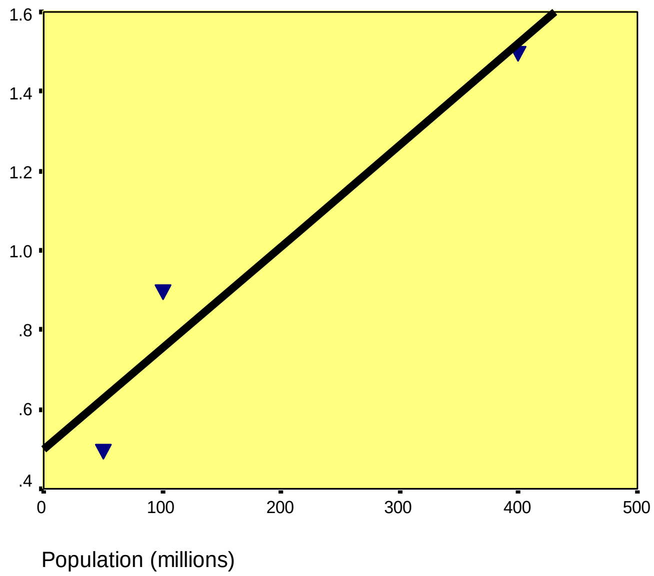


The pre-industrial population growth is usually measured for 100–200 year periods. If we split 450 years in 3 equal intervals, we will get the following picture (see Table 5.2):

Table 5.2. Dynamics Produced by the Pure Growth Phase Lengthening Mechanism

Period	*Years*	*Population at the beginning of cycle (millions)*	*Average population growth rate during 150-year period*
1	0 – 150	50	0.5
2	151 – 300	100	0.9
3	301 – 450	400	1.5

Diagram A13. Relationship between Population Size and Growth Rate Produced by the Pure Growth Phase Lengthening Mechanism



NOTE: R = 0.962, R^2 = 0.926, p = 0.088 (1-tailed).

As we see, if higher populations raise the carrying capacity to higher levels, this creates a certain hyperbolic effect, even if the annual growth rates during pure growth phases do not increase, and even if the interphases do not become shorter.

There is some evidence that during the last pre-industrial pure growth phase the carrying capacity was raised in greater proportion than during the previous phase (Table A3):

Table A3. Two Last Pre-Industrial World Population Growth Phases Compared

Growth phases	*Population at the beginning of the phase*	*Population at the end of the phase*	*Population growth achieved during the phase*
1400–1600	**350**	**545**	**55%**
1650–1800	**545**	**900**	**65%**
1650–1850	**545**	**1200**	**120%**

Note that this difference becomes more pronounced if we consider as the end of the last pre-industrial pure growth phase 1850 rather than 1800.[11] In any case, the mechanism under consideration accounts to a very considerable extent for the fact that the last period of uninterrupted world population growth (combining pre-industrial and industrial phases) was longer than the previous period, which in its turn contributed to the hyperbolic population growth trend.

Two 2nd millennium pre-industrial Chinese growth phases about which we know some detail are the Sung and Qing ones[12] (see Table A4):

Table A4. Sung and Qing Pure Growth Phases Compared

Growth phases	*Population at the beginning of the phase (millions)*	*Population at the end of the phase (millions)*	*Population growth achieved during the phase*
1050–1110	**54**	**104.5**	**93.5%**
1740–1850	**150**	**436.3**	**190.8%**

An interesting thing about Sung – Qing comparison is that during the Qing pure growth phase the annual growth rates were even a bit lower (*c.* 1.0 %) than during the Sung one (*c.* 1.1 %), and the hyperbolic effect here was created specifically by the pure-growth-phase-lengthening mechanism.

As we see, for the last pre-Modern pure-growth phases for both China and the world we do observe a certain positive correlation between the population at the beginning of the phase and the increase in carrying capacity achieved during a respective phase. Thus the hyperbolic trend during the last centuries of pre-industrial population growth turns out to be accounted for to some extent by the mechanism under consideration. Note, however, that the respective proportion does not appear to be quadratic, but is rather linear. Thus, though the respective mechanism appears to contribute to the appearance of hyperbolic trend in the population growth in the last centuries of the pre-industrial history, this contribution does not appear to be very high (unlike the one of the mechanism, which we will discuss next).

Interphase Shortening Mechanism

Let us now further re-formulate compact macromodel logic in the following way: the higher the population at the beginning of an interphase, the less time it will take it to start a new series of innovations resulting in a new pure growth

[11] This makes sense, as the world population growth in 1800–1850 resulted mostly from the regions (first of all East Asia), where industrial revolution and demographic transition had not started yet.

[12] As has been mentioned above, due to the defectiveness of the Ming statistics, no such detail is known for the Ming phase.

phase. The empirical evidence appears to support this hypothesis (see Table A5):

Table A5. Interphase Characteristics

China			**The World**		
Interphase	*Population at the beginning of interphase, millions*	*Interphase length*	*Interphase*	*Population at the beginning of interphase, millions*	*Interphase length*
13 – 1050 CE	59.85	1037	100 – 1000	180	900
1110 – 1500 (?)	104.5	390	1200 – 1400	360	200
1580 – 1740	150	160	1600 – 1650	545	50
R	0.963		R	0.934	
R^2	0.92		R^2	0.87	
p (1-tailed)	0.087		p (1-tailed)	0.088	

As we see, we seem to observe a relationship between the population at the beginning of interphase and the interphase length which is close to the inverse quadratic, *i.e.*, the increase in population during the pure growth phase by factor of X leads to the decrease of the subsequent interphase X^2 times (in fact, in most cases even more). Actually, this is just what the compact macromodels' logic suggests: the innovation rate is assumed to be proportional to the population size and technology level. Thus, a population twice as large at the beginning of interphase B (t_B) as compared to the one at the beginning of interphase A (t_A) implies that the technology level at t_B was twice as high as at t_A. Thus, we have grounds to predict that it will take twice as large population having twice as high technology a 4 times smaller period ($2^2 = 4$) to accumulate the same amount of innovations necessary to initiate a new pure growth phase.

We will now model what the contribution of this mechanism to the hyperbolic population growth trend will be.

In our model the first pure growth phase starts in 300 BCE from a 100 million level. During every pure growth phase the population grows with 0.7% annual rate for 100 years, thus doubling within a century. The first interphase is assumed to be 800 years. The length of each subsequent interphase is inversely related to the square of the population growth during the preceding pure growth phase. During each growth phase as the population increases twice, each subsequent interphase becomes shorter than the preceding one by a factor of four. This results in the following dynamics (see Diagrams A14–15 and Table A6):

Diagram A14. Dynamics Produced by the Interphase Shortening Mechanism

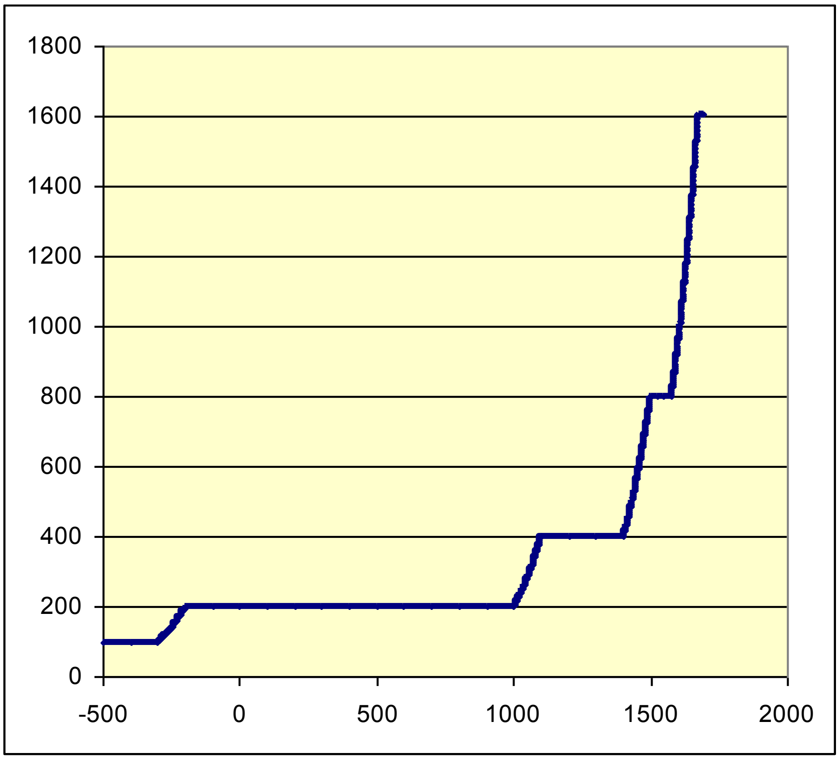
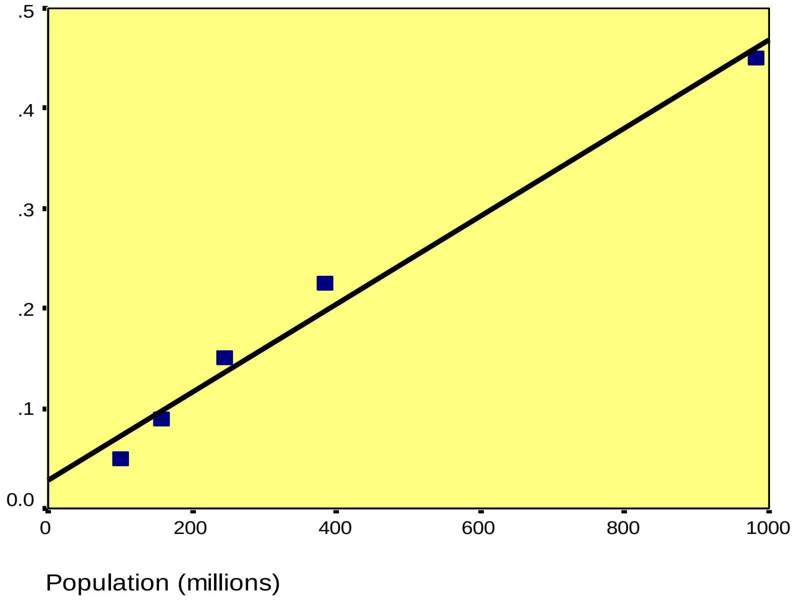


Table A6. Dynamics Produced by the Interphase Shortening Mechanism

Cycle	*Years*	*Population at the beginning of cycle (millions)*	*Average population growth rate during subsequent cycle (growth phase + subsequent inter-phase) (%%)*	*Interphase Length*
1	–300–1000	100	0.054	1200
2	1001–1400	200	0.175	300
3	1401–1575	400	0.4	75
4	1576–1695	800	0.59	18.75

Diagram A15. Relationship between Population Size and Growth Rate Produced by the Interphase Shortening Mechanism



NOTE: R = 0.992, R^2 = 0.984, p = 0.001.

As we see, the interphase shortening mechanism produces a rather strong hyperbolic effect, and we believe that it had the most important contribution to the creating of the pre-industrial world population hyperbolic growth trend. However, it accounts for the increase in the population growth rates during each subsequent developmental cycle (whereas each subsequent developmental cycle occurred on a significantly higher population level); however, it cannot account for the increase in population growth rate during each subsequent pure growth phase, whereas such a trend is also observed (see Table A7):

Table A7. Trend towards the Increase in the World Population Growth Rate during Each Subsequent Pure Growth Phase

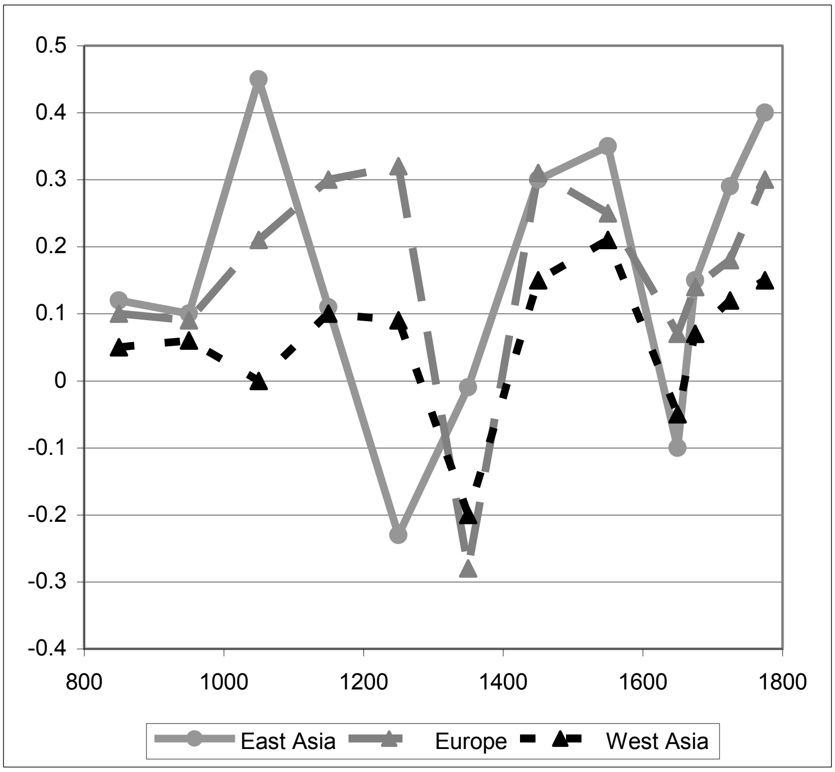
Pure growth phase	*Average population growth rate (%%)*
–500 – 100	0.1
950 – 1200	0.15
1400 – 1600	0.24
1650 – 1850	0.4

This trend appears to be augmented by the effects of the *"increasing synchronization of pure growth phase" mechanism*, which we shall discuss in the next section of this appendix.

"Increasing synchronization of pure growth phases" mechanism

An important feature of the World System history is the increasing synchronization of the growth and decline phases in the various World System centers demonstrated recently by Chase-Dunn *et al.* (2003), as well as by Hall and Turchin (2003) (see Diagram A16):

Diagram A16. Growing Phase Synchronization of the World System (population rate of change dynamics in 3 World System regions, 800 – 1800 CE)



NOTE: Adapted from Hall and Turchin 2003: 15.

There are grounds to believe that this increasing synchronization of pure growth phases was caused to a considerable extent just by the World System population growth. The higher is the population of the World System regions, the more contacts they will have, the faster the innovations will spread, and the higher will be the growth phase synchronization.

What will be the impact of this increasing growth phase synchronization on the population dynamics? It will result in exactly a hyperbolic effect – the average population growth rate within every subsequent growth phase, occurring at the population level higher than during the previous one, will be also higher.

Let us model this effect. We assume that the World System population at the beginning is 100 million and it consists of 4 regions comprising 25 million each. During each growth phase the carrying capacity grows twice; as a result the population of each region grows twice at constant annual rate of 0.5%. During the first phase the synchronization takes 800 years. During each subsequent developmental cycle the synchronization period is assumed to be inversely proportional to the population level reached during the previous cycle. As a result, if the population grows twice during the given cycle, the synchronization period during the subsequent period will shorten twice. This will result in the following dynamics (see Table A8):

Table A8. Dynamics Produced by the "Increasing Synchronization of Pure Growth Phase" Mechanism

Synchronization period (years)	*Population at the beginning of growth phase (millions)*	*Average annual growth rate (%%)*
800	100	0.0625
400	200	0.125
200	400	0.25
100	800	0.5

As we see, up to the complete synchronization this gives a very considerable hyperbolic effect.

Let us model the combined impact of the three above described hyperbolic growth mechanisms (different from the one of the demographic transition first phase).

In this model the World System is assumed to consist of four regions with equal population. The first growth phase accounted for by the model starts in 650 BCE from 120 million level. During the growth phases regional populations grow at 0.4% rate. The first interphase is assumed to be 600 years. Interphase lengths are inversely proportional to the square of population; increase in the pure growth phases in each region is lineally proportional to the population increase during the previous phase. The first synchronization period is 800 years. The length of a synchronization period is inversely proportional to the population growth during the previous cycle. This model generates the following dynamics (see Diagram A17 and Table A8):

Diagram A17. Dynamics Produced by the Combined Action of Mechanisms of "Pure Growth Phase Lengthening", "Interphase Shortening", and "Increasing Pure Growth Phase Synchronization

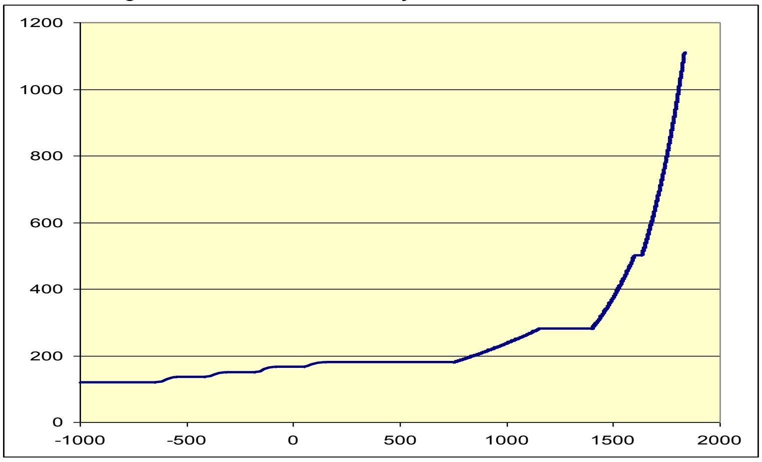


Table A8.

Period	*Average world population growth rate*	*World population at the beginning of the period*	*World population at the end of the period*	*Regional pure growth phase lengths*
–650 – –550	0.1%	120	135	100
–550 – –417	0	135	135	
–417 – –317	0.1%	135	150	100
–317 – –184	0	150	150	
–184 – –84	0.1%	150	165	100
–84 – 50	0	165	165	
50 – 150	0.1%	165	180	100
150–750	0	180	180	
750–1150	0.11%	180	280	110
1150–1400	0	280	280	
1401–1600	0.29%	280	500	145
1601–1635	0	500	500	
1636–1835	0.4%	500	1110	200

This model demonstrates an especially close fit with the observed data. The main discrepancy is produced by the fact that this model does not account for the hyperbolic trends within the pure growth phases. This trend is especially pronounced within the last pre-industrial pure growth phase (1650–1850, for which we, incidentally, have the most accurate data within the pre-industrial epoch) (see Table A9):

Table A9. World Population Dynamics, 1650–1850 (according to McEvedy and Jones [1978] and Kremer [1993])

Period	*Population at the beginning of the period (millions)*	*Average world population growth rate during the period (%%)*
1650–1700	545	0.225
1700–1750	610	0.332
1750–1800	720	0.446
1800–1850	900	0.575

Is it possible to account for such a trend without dropping the assumption that the average population growth rate within any pre-industrial pure growth phase cannot exceed 0.4%? Yes, this is possible if we take into account two more mechanisms of hyperbolic growth. We start with the Innovation Diffusion Mechanism.

Innovation diffusion mechanism

Its logic can be formulated as follows:

The diffusion of a carrying capacity increasing technology within a world system with a stagnant population will result in a quasi-hyperbolic demographic growth trend (even if the annual population growth rate after this technology introduction remains constant) due to the rise of the proportion of the growing population.

Let us model the impact of this mechanism on population dynamics using the following model.

In this model the World System consists of 4 regions, each of which consists of 4 zones. At the beginning all the zones have equal populations. A new technology starts to be introduced in all the 4 regions. It raises the carrying capacity so that it allows the population growth at 0.6% rate for 200 years. Assume that the innovation is not introduced immediately in all the zones; it is implemented during each phase in one more zone of each World System region. This will result in the following dynamics (see Table A10):

Table A10. Dynamics Produced by the Innovation Diffusion Mechanism

Sub-Phase	*Years*	*Zones where innovations are implemented*	*Annual population growth rate at the beginning of sub-phase (%%)*	*Population at the beginning and at the end of sub-phase (millions)*
1	1–50	25%	0.15	545–587
2	50–100	50%	>0.3	587–681
3	100–150	75%	>0.45	681–852
4	150–200	100%	0.6	852–1150

NOTE: average annual growth rate during the phase = 0.4%.

Note that within this model we arrive at the hyperbolic growth within the pure growth phase, though the average population growth rate during the whole phase remains 0.4%.

Differential growth mechanism

Note that within the model above a quasi-hyperbolic growth starts immediately after the carrying capacity increasing technology is introduced in 25% of the world system zones and is observed within 50 years of Phase 1, even though this technology is assumed to only spread to the next belt zones in Phase 2. Thus, though the number of zones where the new technology is introduced remains constant, and the population in these zones increases at constant rate, the World System population during 50 years of sub-phase 1 experiences a quasi-hyperbolic growth simply due to the differential growth mechanism.

Its logic can be formulated as follows:

If a new carrying capacity increasing technology is introduced only in one of the World System zone (A) (with all the other zones having stagnant population), it does not diffuse to the other zones, and the annual population growth in Zone A remains constant, the World System population growth will be characterized by a quasi-hyperbolic trend, due to the increase in the portion of the population that is growing.

Let us model this mechanism impact on population dynamics using the following model:

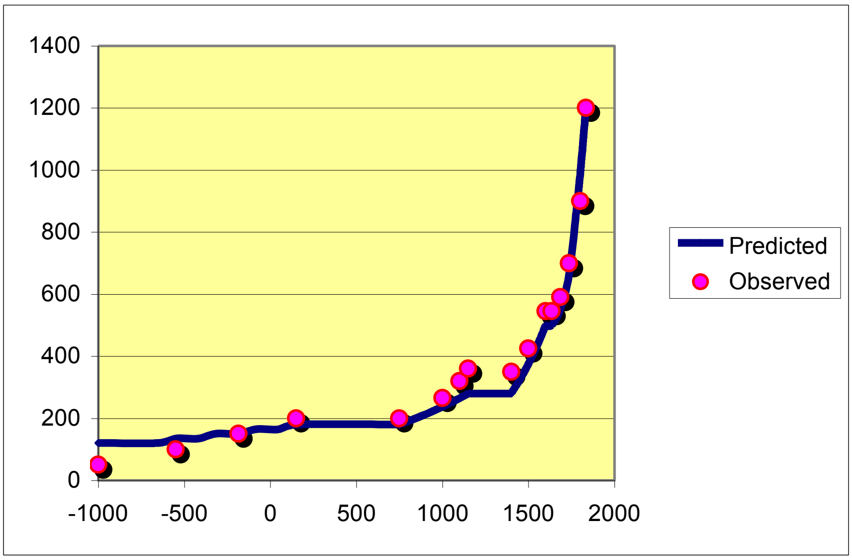
In this model zones comprising 25% of the world introduce innovations and their population starts increasing at 2.05% growth rate (thus, growing *c.* 50% every 20 years). The population of the rest of the world does not grow. This results in the following dynamics (Table A11):

Table A11. Dynamics Produced by the Differential Growth Mechanism

Year	*Population of innovation zones (millions)*	*Population of the world (millions)*	*World population growth rate (%%)*
1	100	400	0.5125
20	150	450	0.683
40	225	525	0.879
60	337.5	637.5	1.085
80	506.25	806.25	1.287
100	759.375	1059.375	1.47

Introducing the effect of the last two mechanisms for the last phase (note that the average annual growth rate in all the regions during the last pure growth phase still remains 0.4%) we arrive at the following dynamics showing the closest fit with the observed data (see Diagram A18):

Diagram A18.[13] Dynamics Produced by the Combined Effect of the Considered Mechanisms



Conclusion

We believe that the 5 mechanisms of hyperbolic growth suggested in this part of our book (in addition to the one of the first phase of demographic transition) account quite satisfactorily for the hyperbolic trend observed for the pre-industrial world population without making a counter-factual assumption that the growth rate of world populations tended to increase with each subsequent cycle. Hence, this model does not contradict the available data suggesting the absence of any significant world trend toward the growth of life expectancies in the pre-industrial era (as we remember, these data rather suggest a weak opposite trend).

Of course, the hyperbolic growth generated by our model is somewhat imperfect in that it is rather different from the one generated be simple hyperbolic growth models; but in this it is similar to the one observed in the historical record of pre-industrial world – all the main deviations from hyperbolic growth turn out to be totally regular phenomena predicted by the last model.

[13] Note that our model predicts a short (*c.*12.5 years) intercycle at the end of the last pre-industrial pure growth phase around 1850 (not reflected in this diagram). There are some reasons to expect that this interphase in the world population growth actually existed, *i.e.*, in 1863 the world population was not higher than in 1851 (due to enormous population losses in China during the Taiping rebellion and accompanying episodes of internal [as well as external] warfare [see, *e.g.*, Nepomnin 2005; Korotayev, Malkov, and Khaltourina 2006b]).

However this does not deny the value of compact mathematical models of the World System growth that describe rather accurately the overall shape of millennial trends and account for their most basic "Kuznetsian" mechanism and logic: more people – more potential inventors – faster technological growth – the carrying capacity of the Earth grows faster – faster population growth – more people – more potential inventors – faster technological growth, and so on, whereas, as we could see at the very beginning of this monograph, such a positive nonlinear feedback finally just produces hyperbolic growth dynamics.