Introduction¹

Human society is a complex nonequilibrium system that changes and develops constantly. Complexity, multivariability, and contradictoriness of social evolution lead researchers to a logical conclusion that any simplification, reduction, or neglect of the multiplicity of factors leads inevitably to the multiplication of error and to significant misunderstanding of the processes under study. The view that any simple general laws are not observed at all with respect to social evolution has become totally predominant within the academic community, especially among those who specialize in the Humanities and who confront directly in their research all the manifold unpredictability of social processes.

A way to approach human society as an extremely complex system is to recognize differences of abstraction and time scale between different levels. If the main task of scientific analysis is to detect the main forces acting on systems so as to discover fundamental laws at a sufficiently coarse scale, then abstracting from details and deviations from general rules may help to identify measurable deviations from these laws in finer detail and shorter time scales. Modern achievements in the field of mathematical modeling suggest that social evolution can be described with rigorous and sufficiently simple macrolaws. Our goal, at this stage, is to discuss a family of mathematical models whose greater specification leads to measurable variables and testable relationships.

Tremendous successes and spectacular developments in physics (especially, in comparison with other sciences) were, to a considerable degree, connected with the fact that physics managed to achieve a synthesis of mathematical methods and subject knowledge. Notwithstanding the fact that already in the classical world physical theories achieved a rather high level, it was in the modern era that the introduction of mathematics made it possible to penetrate deeper into the essence of physical laws, laying the ground for the scientific-technological revolution. However, such a synthesis was not possible without one important condition. Mathematics operates with forms and numbers, and, hence, the physical world had to be translated into the language of forms and numbers. It demanded the development of effective methods for measuring physical values and the introduction of scales and measures. Starting with the simplest variables – length, mass, time – physicists learned how to measure

charge, viscosity, inductance, spin and many other variables, which are necessary for the development of the physical theory of value.

In an analogous way, a constructive synthesis of the social sciences with mathematics calls for the introduction of adequate methods for the measurement of social variables. In the social sciences, as in physics, some variables can be measured relatively easily, while the measurement of some other variables needs additional research and even the development of auxiliary models.

One social variable that is relatively well accessible to direct measurement is population size. That is why it is not surprising that the field of demography attracts the special attention of social scientists, as it suggests some hope for the development of quantitatively based scientific theories. It is remarkable that the penetration of mathematical methods into biology began, to a considerable extent, with the description of population dynamics.

The basic measurability of data is quite evident here; what is more, the basic equation for the description of demographic dynamics is also rather evident, as it stems from the conservation law:

$$\frac{dN}{dt} = B - D, \tag{0.1}$$

where N is the number of people, B is the number of births, and D is the number of deaths in the unit of time. However, at the microlevel it turns out that both the number of deaths and number of births depend heavily on a huge number of social parameters, including the "human factor" – decisions made by individual people that are very difficult to formalize.

In addition to this, equation (0.1) does not take into account the spatial movement of people; hence, it should be extended:

$$\frac{\partial N}{\partial t} = B - D - \text{div}\mathbf{J}, \tag{0.1}$$

where vector **J** corresponds to the migration current. In this case the problem becomes even more complicated, as migration processes are even more likely to be influenced by external factors.

That is why any formal description of demographic processes at the microlevel confronts serious problems associated first of all with the lack of sufficient research on formal social laws connecting economic, political, ethical and other factors that affect individual and small group (*e.g.*, household or nuclear family) behavior. Thus, at the moment the only available approach is macrolevel description that does not go into the fine details of demographic processes and deIntroduction 7

scribes dynamics of very large human populations, which is influenced by the human factor at a significantly coarser level of abstraction and on a longer time-scale.

Biological processes of birth and death are characteristic not only of people, but also of any animals. That is why a rather natural step is to try to describe demographic models using population models developed within biology (see, *e.g.*, Riznichenko 2002).

The basic model describing animal population dynamics is the logistic model, suggested by Verhulst (1838):

$$\frac{dN}{dt} = rN(1 - \frac{N}{K}), \qquad (0.2)$$

which can be also presented in the following way:

$$\frac{dN}{dt} = (a_1 N) - (a_2 N + bN^2), \tag{0.3}$$

where a_1N corresponds to the number of births B, and $a_2N + bN^2$ corresponds to the number of deaths in equation (0.1); r, K, a_1 , a_2 , b are positive coefficients connected between themselves by the following relationships:

$$r = a_1 - a_2 \quad \text{and} \quad b = \frac{r}{K}, \tag{0.4}$$

The logic of equation (0.3) is as follows: fertility a_I is a constant; thus, the number of births $B = a_I N$ is proportional to the population size, natural death rate a_2 is also considered to be constant, whereas quadratic addition bN^2 in expression for full number of deaths $D = a_1 N + bN^2$ appears due to the resource limitation, which does not let population grow infinitely. Coefficient b is called the coefficient of interspecies competition.

As a result, the population dynamics described by the logistic equation has the following characteristics. At the beginning, when the size of the animal population size is low, we observe an exponential growth with exponent $r = a_1 - a_2$. Then, as the ecological niche is being filled, the population growth slows down, and finally the population comes to the constant level K.

The value of parameter K, called the *carrying capacity of an ecological niche for the given population*, is of principal importance. This value deter-

mines the equilibrium state in population dynamics for the given resource limitations and controls the limits of its growth.

Another well known population dynamics model is Lotka – Volterra one (Lotka 1925; Volterra 1926), denoted also as the "prey – predator" model. It describes dynamics of populations of two interacting species, one of which constitutes the main food resource for the other, and consists of two equations of type (0.1):

$$\frac{dx}{dt} = Ax - Bxy$$

$$\frac{dy}{dt} = Cxy - Dy,$$
(0.5)

where x is the size of the prey population, y is the size of the predator population; A, B, C, D are coefficients.

This model, like (0.2), assumes that the number of prey births is proportional to their population size. The number of predator deaths is also proportional to their population size. As regards prey death rates and predator fertility rates, we are dealing here with a system effect. Prey animals are assumed to die mainly because of contacts with predators, whereas the predator fertility rates depend on the availability of food – prey animals. The model assumes that the average number of contacts between prey animals and predators depends mainly on the size of both populations and suggests expression Bxy for the number of prey deaths and Cxy for the number of predator births.

This model generates a cyclical dynamics. The growth of the prey population leads to the growth of the predator population; the growth of predator population leads to the decrease of the prey animal number; decrease of the prey population leads to the decrease of the predators' number; and when the number of predators is very small, the prey population can grow very rapidly.

The population models described above are used very widely in biological research. It seems reasonable to suppose that, as humans have a biological nature, some relations similar to the ones described above, or their analogues could be valid for humans too.

In deep prehistory, when human ancestors did not differ much from animals, models (0.2) - (0.4) may have been applied to them without any significant reservations. However, with the appearance of a new human environment, the sociotechnological one, the direct application of those models does not appear to be entirely adequate. In particular, model (0.2) assumes carrying capacity to be determined by exogenous factors; however, human history shows that over the course of time the carrying capacity of land has tended to increase in a rather

significant way. Hence, in long-range perspective carrying capacity cannot be assumed to be constant and determined entirely by exogenous conditions. Humans are capable of transforming those conditions affecting carrying capacity.

As regards model (0.4), it has an extremely limited applicability to humans in its direct form, as humans learned how to defend themselves effectively from predators at very early stages of their evolution; hence, humans cannot function as "prey" in this model. On the other hand, humans learned how not to depend on the fluctuations of prey animals populations, hence, they cannot function as predators because in model (0.4) predators are very sensitive to the variations of prey animal numbers (This model could still have some limited direct applicability to a very few cases of highly specialized hunters).

However, model (0.4) may find a new non-traditional application in demographic models. In particular it may be applied to the description of demographic cycles that have been found in historical dynamics of almost all the agrarian societies, for which relevant data are available. The population plays here the role of "prey", whereas the role of "predator" belongs to sociopolitical instability, internal warfare, famines and epidemics whose probability increases when an increasing population approaches the carrying capacity ceiling (for detail see, e.g., Korotayev, Malkov and Khaltourina 2005: 211–54). Demographic cycles are by themselves a very interesting subject for mathematical research, and they have been studied rather actively in recent years (Usher 1989; Chu and Lee 1994; Malkov and Sergeev 2002, 2004; Malkov et al. 2002; Malkov 2002, 2003, 2004; Malkov, Selunskaja, and Sergeev 2005; Turchin 2003, 2005a, 2005b; Turchin and Korotayev 2006; Nefedov 2002a; 2004; Korotayev, Malkov and Khaltourina 2005 etc.)

As is well known in complexity studies, chaotic dynamics at the microlevel can generate a highly deterministic macrolevel behavior (*e.g.*, Chernavskij 2004). To describe behavior of a few dozen gas molecules in a closed vessel we need very complex mathematical models; and these models would still be unable to predict long-run dynamics of such a system due to inevitable irreducible chaotic components. However, the behavior of zillions of gas molecules can be described with extremely simple sets of equations, which are capable of predicting almost perfectly the macrodynamics of all the basic parameters (just because of chaotic behavior at microlevel). Of course, one cannot fail to wonder whether a similar set of regularities would not also be observed in the human world too. That is, cannot a few very simple equations account for an extremely high proportion of all the macrovariation with respect to the largest possible social system – the World System?